

Quantum Gravitational Effects on Massive Fermions during Inflation I

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ABSTRACT

We compute the one loop graviton contribution to the self-energy of a very light fermion on a locally de Sitter background. This result can be used to study the effect that a small mass has on the propagation of fermions through the sea of infrared gravitons generated by inflation. We employ dimensional regularization and obtain a fully renormalized result by absorbing all divergences with BPHZ counterterms. An interesting technical aspect of this computation is the need for two noninvariant counterterms owing to the breaking of de Sitter invariance by our gauge condition.

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1 Introduction

In this paper we compute and renormalize the one loop quantum gravitational corrections to the self-energy of very light fermions on a locally de Sitter background. The physical motivation for this exercise is to facilitate a later study of how inflationary gravitons affect fermions and, in particular, the contrast between the case of exactly massless fermions and those with a small mass. Nonzero mass introduces two competing effects: it changes how fermions propagate and it also alters how they interact with gravity. The first of these changes tends to suppress the effects of inflationary gravitons because it makes the fermion wave function oscillate so that interactions at different times tend to cancel. However, the new interaction enhances the effect of inflationary gravitons because it does not fall off with time.

The current work can be seen as complementing two previous studies of massless fermions on de Sitter. In both cases the technique was to compute the one loop fermion self-energy $-i[\Sigma_j](x; x')$ and then use it to solve the quantum-corrected Dirac equation for fermion mode functions,

$$\sqrt{-g} i \not{D}_{ij} \psi_j(x) - \int d^4 x' [{}_i \Sigma_j](x; x') \psi_j(x') = 0 . \quad (1)$$

The first model results from Yukawa coupling the fermion to a massless, minimally coupled (MMC) scalar on a nondynamical de Sitter background [3]. The second model consists of the fermion with dynamical gravity on de Sitter background [1]. Powers of the inflationary scale factor $a = e^{Ht}$ are crucial for understanding the results in both cases. The self-energy from the $\phi \bar{\psi} \psi$ interaction of the first model grows like $a \ln(a)$ relative to the classical term. The result fermion mode functions behave as if they had a growing mass. The interactions of the second model all possess derivatives — for example, $\partial h \bar{\psi} \psi$ — which limit the induced self-energy to grow no faster than $\ln(a)$ relative to the classical term. The resulting fermion mode functions behave as if they had a growing field strength, which could be understood as the random walk that fermions take under buffeting from the sea of inflationary gravitons [1, 2]. Although the effect from gravitons is smaller than that from massless scalars, *it is universal*, independent of assumptions about the existence or couplings of unnaturally light scalars. It is even conceivable that the graviton effect might, in a more complicated model, lead to baryogenesis during inflation.

What we expect for massive fermions in dynamical gravity is that the absence of derivatives in the $m h \bar{\psi} \psi$ interaction will cause the self-energy to

grow like $a \ln(a)$ relative to the classical kinetic term, and like $\ln(a)$ relative to the classical mass term. When the classical mass is large (relative to the Hubble parameter) we expect at most a small enhancement of the fermion field strength. When the classical mass is small, classical dynamics are mostly controlled by the kinetic term and we expect the quantum correction to have a much larger proportional impact. One might intuitively expect the crossover to come for fermion masses near the Hubble parameter. However, we shall specialize to the case of very light fermions, both because this is where the largest effects should occur, and because expanding in the fermion mass makes an enormous simplification in the computation.

This work also deserves a place in the growing list of studies of quantum infrared effects during inflation. Among these are:

- The effects of self-interacting, MMC scalars on nondynamical de Sitter background [45, 46, 47, 48];
- The effects of a charged, MMC scalar on nondynamical de Sitter background [81, 83];
- The effects of a nonlinear sigma model on nondynamical de Sitter background [51, 52];
- The effects of a MMC scalar on gravitons on de Sitter background [49, 50];
- The effects of gravitons on a MMC scalar on de Sitter background [43]; and
- The effects of gravitons on interacting conformal matter on de Sitter background [53, 54].

It should also be noted that the series of leading infrared logarithms can be summed for scalar potential models using the stochastic technique of Starobinsky and Yokoyama. The same resummation can be achieved for Yukawa theory, and for scalar QED, but it has so far not been accomplished for either the nonlinear sigma model, or for quantum gravity. Each fully renormalized quantum gravitational result is an important piece of “data” in the search for such a resummation.

Dirac + Einstein is not perturbatively renormalizable [5], however, ultraviolet divergences can always be absorbed in the BPHZ sense [6, 7, 8, 9]. A

widespread misconception exists that no valid quantum predictions can be extracted from such an exercise. This is false: while nonrenormalizability does preclude being able to compute *everything*, that not the same thing as being able to compute *nothing*. The problem with a nonrenormalizable theory is that no physical principle fixes the finite parts of the escalating series of BPHZ counterterms needed to absorb ultraviolet divergences, order-by-order in perturbation theory. Hence any prediction of the theory that can be changed by adjusting the finite parts of these counterterms is essentially arbitrary. However, loops of massless particles make nonlocal contributions to the effective action that can never be affected by local counterterms. These nonlocal contributions typically dominate the infrared. Further, they cannot be affected by whatever modification of ultraviolet physics ultimately results in a completely consistent formalism. As long as the eventual fix introduces no new massless particles, and does not disturb the low energy couplings of the existing ones, the far infrared predictions of a BPHZ-renormalized quantum theory will agree with those of its fully consistent descendant.

It is worthwhile to review the vast body of distinguished work that has exploited this fact. The oldest example is the solution of the infrared problem in quantum electrodynamics by Bloch and Nordsieck [10], long before that theory's renormalizability was suspected. Weinberg [11] was able to achieve a similar resolution for quantum gravity with zero cosmological constant. The same principle was at work in the Fermi theory computation of the long range force due to loops of massless neutrinos by Feinberg and Sucher [12, 13]. Matter which is not supersymmetric generates nonrenormalizable corrections to the graviton propagator at one loop, but this did not prevent the computation of photon, massless neutrino and massless, conformally coupled scalar loop corrections to the long range gravitational force [14, 15, 16, 17]. More recently, Donoghue [18, 19] has touched off a minor industry [20, 21, 22, 51, 52] by applying the principles of low energy effective field theory to compute graviton corrections to the long range gravitational force. Our analysis exploits the power of low energy effective field theory in the same way, differing from the previous examples only in the detail that our background geometry is locally de Sitter rather than flat.¹

That summarizes why the exercise we have undertaken is both valid and interesting. In the next section we work out the fermionic part of the Feynman rules for Dirac + Einstein. Section 3 is devoted to the issues associated

¹For another recent example in a nontrivial cosmology see [25].

with the graviton propagator. A major complication concerns the impossibility of employing an average, de Sitter invariant gauge condition [26, 27]. We give a short review of the complex literature on this issue. Then we introduce a noninvariant gauge fixing term, isolate the subgroup of de Sitter transformations that it respects, and present the gauge-fixed graviton propagator. The section closes with a discussion of the BPHZ counterterms necessary for our computation. In section 4 we evaluate the contributions from diagrams involving a single 4-point interaction. In section 5 we evaluate the more difficult contributions which involve two 3-point interactions. Renormalization is accomplished in section 6, and our conclusions are given in section 7.

2 Fermions in Quantum Gravity

The coupling of gravity to particles with half integer spin is usually accomplished by shifting the fundamental gravitational field variable from the metric $g_{\mu\nu}(x)$ to the vierbein $e_{\mu m}(x)$.² Greek letters stand for coordinate indices and Latin letters denote Lorentz indices, and both kinds of indices take values in the set $\{0, 1, 2, \dots, (D-1)\}$. One recovers the metric by contracting two vierbeins into the Lorentz metric η^{bc} ,

$$g_{\mu\nu}(x) = e_{\mu b}(x)e_{\nu c}(x)\eta^{bc} . \quad (2)$$

The coordinate index is raised and lowered with the metric ($e^\mu{}_b = g^{\mu\nu}e_{\nu b}$), while the Lorentz index is raised and lowered with the Lorentz metric ($e_\mu{}^b = \eta^{bc}e_{\mu c}$). We employ the usual metric-compatible and vierbein compatible connections,

$$g_{\rho\sigma;\mu} = 0 \implies \Gamma^\rho_{\mu\nu} = \frac{1}{2}g^{\rho\sigma}(g_{\sigma\mu,\nu} + g_{\nu\sigma,\mu} - g_{\mu\nu,\sigma}) , \quad (3)$$

$$e_{\beta b;\mu} = 0 \implies A_{\mu cd} = e^\nu{}_c(e_{\nu d,\mu} - \Gamma^\rho_{\mu\nu}e_{\rho d}) . \quad (4)$$

Fermions also require gamma matrices, γ^b_{ij} . The anti-commutation relations,

$$\{\gamma^b, \gamma^c\} \equiv (\gamma^b\gamma^c + \gamma^c\gamma^b) = -2\eta^{bc}I , \quad (5)$$

imply that only fully anti-symmetric products of gamma matrices are actually independent. The Dirac Lorentz representation matrices are such an anti-

²For another approach see [28].

symmetric product,

$$J^{bc} \equiv \frac{i}{4}(\gamma^b \gamma^c - \gamma^c \gamma^b) \equiv \frac{i}{2} \gamma^{[b} \gamma^{c]} . \quad (6)$$

They can be combined with the spin connection (4) to form the Dirac co-variant derivative operator,

$$\mathcal{D}_\mu \equiv \partial_\mu + \frac{i}{2} A_{\mu cd} J^{cd} . \quad (7)$$

Other identities we shall often employ involve anti-symmetric products,

$$\gamma^b \gamma^c \gamma^d = \gamma^{[b} \gamma^c \gamma^{d]} - \eta^{bc} \gamma^d + \eta^{db} \gamma^c - \eta^{cd} \gamma^b , \quad (8)$$

$$\gamma^b J^{cd} = \frac{i}{2} \gamma^{[b} \gamma^c \gamma^{d]} + \frac{i}{2} \eta^{bd} \gamma^c - \frac{i}{2} \eta^{bc} \gamma^d . \quad (9)$$

We shall also encounter cases in which one gamma matrix is contracted into another through some other combination of gamma matrices,

$$\gamma^b \gamma_b = -DI , \quad (10)$$

$$\gamma^b \gamma^c \gamma_b = (D-2) \gamma^c , \quad (11)$$

$$\gamma^b \gamma^c \gamma^d \gamma_b = 4\eta^{cd} I - (D-4) \gamma^c \gamma^d , \quad (12)$$

$$\gamma^b \gamma^c \gamma^d \gamma^e \gamma_b = 2\gamma^e \gamma^d \gamma^c + (D-4) \gamma^c \gamma^d \gamma^e . \quad (13)$$

The Lagrangian of massive fermions is,

$$\mathcal{L}_{\text{Dirac}} \equiv \bar{\psi} e^\mu_b \gamma^b i \mathcal{D}_\mu \psi \sqrt{-g} - m \bar{\psi} \psi \sqrt{-g} . \quad (14)$$

Because our locally de Sitter background is conformally flat it is useful to rescale the vierbein by an arbitrary function of spacetime $a(x)$,

$$e_{\beta b} \equiv a \tilde{e}_{\beta b} \quad \Longrightarrow \quad e^{\beta b} = a^{-1} \tilde{e}^{\beta b} . \quad (15)$$

Of course this implies a rescaled metric $\tilde{g}_{\mu\nu}$,

$$g_{\mu\nu} = a^2 \tilde{g}_{\mu\nu} \quad \Longrightarrow \quad g^{\mu\nu} = a^{-2} \tilde{g}^{\mu\nu} . \quad (16)$$

The old connections can be expressed as follows in terms of the ones formed from the rescaled fields,

$$\Gamma^\rho_{\mu\nu} = a^{-1} \left(\delta^\rho_\mu a_{,\nu} + \delta^\rho_\nu a_{,\mu} - \tilde{g}^{\rho\sigma} a_{,\sigma} \tilde{g}_{\mu\nu} \right) + \tilde{\Gamma}^\rho_{\mu\nu} \quad (17)$$

$$A_{\mu cd} = -a^{-1} \left(\tilde{e}^\nu_c \tilde{e}_{\mu d} - \tilde{e}^\nu_d \tilde{e}_{\mu c} \right) a_{,\nu} + \tilde{A}_{\mu cd} . \quad (18)$$

Although the massive Dirac is not conformal invariant, we still re-scale fermion fields to make it simpler,

$$\Psi \equiv a^{\frac{D-1}{2}} \psi \quad \text{and} \quad \bar{\Psi} \equiv a^{\frac{D-1}{2}} \bar{\psi} . \quad (19)$$

After performing the above rescalings, the Dirac Lagrangian is

$$\mathcal{L}_{\text{Dirac}} = \bar{\Psi} \tilde{e}^\mu{}_b \gamma^b i \tilde{\mathcal{D}}_\mu \Psi \sqrt{-\tilde{g}} - am \bar{\Psi} \Psi \sqrt{-\tilde{g}} , \quad (20)$$

where $\tilde{\mathcal{D}}_\mu \equiv \partial_\mu + \frac{i}{2} \tilde{A}_{\mu cd} J^{cd}$.

One could follow early computations about flat space background [29, 30] in defining the graviton field as a first order perturbation of the (conformally rescaled) vierbein. However, so much of gravity involves the vierbein only through the metric that it is simpler to instead take the graviton field to be a first order perturbation of the conformally rescaled metric,

$$\tilde{g}_{\mu\nu} \equiv \eta_{\mu\nu} + \kappa h_{\mu\nu} \quad \text{with} \quad \kappa^2 = 16\pi G . \quad (21)$$

We then impose symmetric gauge ($e_{\beta b} = e_{b\beta}$) to fix the local Lorentz gauge freedom, and solve for the vierbein in terms of the graviton,

$$\tilde{e}[\tilde{g}]_{\beta b} \equiv \left(\sqrt{\tilde{g} \eta^{-1}} \right)_\beta{}^\gamma \eta_{\gamma b} = \eta_{\beta b} + \frac{1}{2} \kappa h_{\beta b} - \frac{1}{8} \kappa^2 h_\beta{}^\gamma h_{\gamma b} + \dots \quad (22)$$

It can be shown that the local Lorentz ghosts decouple in this gauge and one can treat the model, at least perturbatively, as if the fundamental variable were the metric and the only symmetry were diffeomorphism invariance [31]. At this stage there is no more point in distinguishing between Latin letters for local Lorentz indices and Greek letters for vector indices. Other conventions are that graviton indices are raised and lowered with the Lorentz metric ($h^\mu{}_\nu \equiv \eta^{\mu\rho} h_{\rho\nu}$, $h^{\mu\nu} \equiv \eta^{\mu\rho} \eta^{\nu\sigma} h_{\rho\sigma}$) and that the trace of the graviton field is $h \equiv \eta^{\mu\nu} h_{\mu\nu}$. We also employ the usual Dirac “slash” notation,

$$\not{V}_{ij} \equiv V_\mu \gamma^\mu_{ij} . \quad (23)$$

It is straightforward to expand all familiar operators in powers of the graviton field,

$$\tilde{e}^\mu{}_b = \delta^\mu{}_b - \frac{1}{2} \kappa h^\mu{}_b + \frac{3}{8} \kappa^2 h^{\mu\rho} h_{\rho b} + \dots , \quad (24)$$

$$\tilde{g}^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^\mu{}_\rho h^{\rho\nu} - \dots , \quad (25)$$

$$\sqrt{-\tilde{g}} = 1 + \frac{1}{2} \kappa h + \frac{1}{8} \kappa^2 h^2 - \frac{1}{4} \kappa^2 h^{\rho\sigma} h_{\rho\sigma} + \dots \quad (26)$$

Applying these identities to the conformally rescaled Dirac Lagrangian gives,

$$\begin{aligned}\mathcal{L}_{\text{Dirac}} = & \bar{\Psi} [i\partial - am] \Psi + \frac{\kappa}{2} \bar{\Psi} [hi\partial - h^{\mu\nu} \gamma_\mu i\partial_\nu - h_{\mu\rho, \sigma} \gamma^\mu J^{\rho\sigma} - amh] \Psi \\ & + \kappa^2 \left\{ \left[\frac{1}{8} h^2 - \frac{1}{4} h^{\rho\sigma} h_{\rho\sigma} \right] \bar{\Psi} i\partial \Psi + \left[-\frac{1}{4} h h^{\mu\nu} + \frac{3}{8} h^{\mu\rho} h_\rho^\nu \right] \bar{\Psi} \gamma_\mu i\partial_\nu \Psi \right. \\ & + \left[-\frac{1}{4} h h_{\mu\rho, \sigma} + \frac{1}{8} h_\rho^\nu h_{\nu\sigma, \mu} + \frac{1}{4} (h_\mu^\nu h_{\nu\rho})_{, \sigma} + \frac{1}{4} h_\sigma^\nu h_{\mu\rho, \nu} \right] \bar{\Psi} \gamma^\mu J^{\rho\sigma} \Psi \\ & \left. - am \left[\frac{1}{8} h^2 - \frac{1}{4} h^{\alpha\beta} h_{\alpha\beta} \right] \bar{\Psi} \Psi \right\} + \mathcal{O}(\kappa^3). \quad (27)\end{aligned}$$

We observe that the rescaled massive fermion propagator in a locally de Sitter background could be derived from the solution of Candelas and Raine [32, 33]. The relation between them is only up to some powers of scale factors,

$$iS[m](x; x')_{C.R.} = (aa')^{-\frac{D-1}{2}} iS[m](x; x'), \quad (28)$$

therefore the conformally re-scaled fermion propagator is,

$$\begin{aligned}iS[m](x; x') = & \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \Gamma\left(\frac{D}{2} - 1\right) (aa')^{\frac{D-1}{2}} \left(ai\mathcal{D} \frac{1}{\sqrt{aa'}} + \sqrt{\frac{a}{a'}} m I \right) \\ & \times \left\{ \frac{\Gamma(\frac{D}{2} - 1 + i\frac{m}{H}) \Gamma(\frac{D}{2} - i\frac{m}{H})}{\Gamma(\frac{D}{2} - 1) \Gamma(\frac{D}{2})} {}_2F_1\left(\frac{D}{2} - 1 + i\frac{m}{H}, \frac{D}{2} - i\frac{m}{H}; \frac{D}{2}; 1 - \frac{y}{4}\right) \left(\frac{I - \gamma^0}{2}\right) \right. \\ & \left. + \frac{\Gamma(\frac{D}{2} - 1 - i\frac{m}{H}) \Gamma(\frac{D}{2} + i\frac{m}{H})}{\Gamma(\frac{D}{2} - 1) \Gamma(\frac{D}{2})} {}_2F_1\left(\frac{D}{2} - 1 - i\frac{m}{H}, \frac{D}{2} + i\frac{m}{H}; \frac{D}{2}; 1 - \frac{y}{4}\right) \left(\frac{I + \gamma^0}{2}\right) \right\}. \quad (29)\end{aligned}$$

where the coordinate interval is $\Delta x^2(x; x') \equiv \|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\delta)^2$ and $i\mathcal{D}$ here is just $a^{-(\frac{D+1}{2})} i\partial a^{(\frac{D-1}{2})}$.

It is useful to recast the solution (29) using the transformation formula of hypergeometric functions [34] and expands out its series expansion,

$$\begin{aligned}iS[m](x; x') = & \frac{\Gamma(\frac{D}{2} - 1)}{4\pi^{\frac{D}{2}}} [i\partial + am] \frac{1}{\Delta x^{D-2}} \\ & + \frac{(H^2 aa')^{\frac{D-1}{2}}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2} - 1) \Gamma(2 - \frac{D}{2}) (i\frac{m}{H})}{\Gamma(1 + i\frac{m}{H}) \Gamma(1 - i\frac{m}{H})} \left[i\partial + \left(\frac{D}{2} - 1\right) iHa\gamma^0 + am \right] \\ & \times \sum_{n=0}^{\infty} \left\{ \frac{\Gamma(n + \frac{D}{2} - 1 + i\frac{m}{H}) \Gamma(n + \frac{D}{2} - 1 - i\frac{m}{H})}{\Gamma(n + \frac{D}{2} - 1) \Gamma(n + 1)} \left[\frac{i\frac{m}{H}}{(n + \frac{D}{2} - 1)} + \gamma^0 \right] \left(\frac{y}{4}\right)^n \right. \\ & \left. - \frac{\Gamma(n + 1 + i\frac{m}{H}) \Gamma(n + 1 - i\frac{m}{H})}{\Gamma(n + 1) \Gamma(n + 3 - \frac{D}{2})} \left[\frac{i\frac{m}{H}}{(n + 1)} + \gamma^0 \right] \left(\frac{y}{4}\right)^{n+2 - \frac{D}{2}} \right\}. \quad (30)\end{aligned}$$

Notice that (30) recovers the fermion propagator in flat spacetime when taking $H = 0$,

$$\lim_{H \rightarrow 0} i[S_j](x; x') = i[S_j]_{\text{cf}}(x; x') + i[S_j]_{\text{fm}}(x; x'), \quad (31)$$

$$i[S_j]_{\text{cf}}(x; x') = \frac{\Gamma(\frac{D}{2}-1)}{4\pi^{\frac{D}{2}}} i\partial_{ij} \frac{1}{\Delta x^{D-2}}, \quad (32)$$

$$i[S_j]_{\text{fm}}(x; x') = \frac{\Gamma(\frac{D}{2}-1)}{4\pi^{\frac{D}{2}}} \frac{ma}{\Delta x^{D-2}}. \quad (33)$$

Here “cf” stands for “conformal” and “fm” stands for “flat spacetime mass”.

Because we only endow fermions very small mass compared with the Hubble parameter, for the computation purpose we simplify the infinite series expansion by only keeping terms at order m ,

$$\begin{aligned} i[S_j](x; x') &= i[S_j]_{\text{cf}}(x; x') + i[S_j]_{\text{fm}}(x; x') \\ &- \left(\frac{m}{H}\right) \frac{(H^2 a a')^{\frac{D}{2}-1}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2}-1)\Gamma(3-\frac{D}{2})}{(2-\frac{D}{2})} [\not{\partial}\gamma^0 + (\frac{D}{2}-1)Ha] \\ &\sum_{n=0}^{\infty} \left\{ \frac{\Gamma(n+\frac{D}{2}-1)}{\Gamma(n+1)} \left(\frac{y}{4}\right)^n - \frac{\Gamma(n+1)}{\Gamma(n+3-\frac{D}{2})} \left(\frac{y}{4}\right)^{n+2-\frac{D}{2}} \right\} + \mathcal{O}(m^2). \end{aligned} \quad (34)$$

Even though the two infinite series expansions tend to cancel out with each other in $D = 4$, the combinations are still finite owing to the divergent factor $\frac{1}{(2-\frac{D}{2})}$. In addition, they can not be expressed in closed form such as Hankel function. The facts which we mentioned above complicate the whole computation.

We now represent the various interaction terms in (27) as vertex operators acting on the fields. At order κ the interactions involve fields, $\bar{\Psi}_i$, Ψ_j and $h_{\alpha\beta}$, which we number “1”, “2” and “3”, respectively. Each of the three interactions can be written as some combination $V_{ij}^{\alpha\beta}$ of tensors, spinors and a derivative operator acting on these fields. For example, the first interaction is,

$$\frac{\kappa}{2} h \bar{\Psi}_i \not{\partial} \Psi = \frac{\kappa}{2} \eta^{\alpha\beta} i \not{\partial}_{2ij} \times \bar{\Psi}_i \Psi_j h_{\alpha\beta} \equiv V_{1ij}^{\alpha\beta} \times \bar{\Psi}_i \Psi_j h_{\alpha\beta}. \quad (35)$$

Hence the 3-point vertex operators are,

$$\begin{aligned} V_{1ij}^{\alpha\beta} &= \frac{\kappa}{2} \eta^{\alpha\beta} i \not{\partial}_{2ij} \quad , \quad V_{2ij}^{\alpha\beta} = -\frac{\kappa}{2} \gamma_{ij}^{(\alpha} i \partial_2^{\beta)} \\ V_{3ij}^{\alpha\beta} &= -\frac{\kappa}{2} (\gamma^{(\alpha} J^{\beta)\mu})_{ij} \partial_{3\mu} \quad , \quad V_{4ij}^{\alpha\beta} = -\frac{\kappa}{2} a m \eta^{\alpha\beta} I_{ij}. \end{aligned} \quad (36)$$

#	Vertex Operator	#	Vertex Operator
1	$\frac{1}{8}\kappa^2\eta^{\alpha\beta}\eta^{\rho\sigma}i\partial_{2ij}$	6	$\frac{1}{8}\kappa^2\eta^{\alpha\rho}(\gamma^\mu J^{\beta\sigma})_{ij}\partial_{4\mu}$
2	$-\frac{1}{4}\kappa^2\eta^{\alpha\rho}\eta^{\sigma\beta}i\partial_{2ij}$	7	$\frac{1}{4}\kappa^2\eta^{\alpha\rho}(\gamma^\beta J^{\sigma\mu})_{ij}(\partial_3 + \partial_4)_\mu$
3	$-\frac{1}{4}\kappa^2\eta^{\alpha\beta}\gamma_{ij}^\rho i\partial_2^\sigma$	8	$\frac{1}{4}\kappa^2(\gamma^\rho J^{\sigma\alpha})_{ij}\partial_4^\beta$
4	$\frac{3}{8}\kappa^2\eta^{\alpha\rho}\gamma_{ij}^\beta i\partial_2^\sigma$	9	$-\frac{1}{8}\kappa^2\eta^{\alpha\beta}\eta^{\rho\sigma}am$
5	$-\frac{1}{4}\kappa^2\eta^{\alpha\beta}(\gamma^\rho J^{\sigma\mu})_{ij}$	10	$\frac{1}{4}\kappa^2\eta^{\alpha\rho}\eta^{\sigma\beta}am$

Table 1: Vertex operators $U_{Iij}^{\alpha\beta\rho\sigma}$ contracted into $\bar{\Psi}_i\Psi_j h_{\alpha\beta}h_{\rho\sigma}$.

The order κ^2 interactions define 4-point vertex operators $U_{Iij}^{\alpha\beta\rho\sigma}$ similarly, for example,

$$\frac{1}{8}\kappa^2 h^2 \bar{\Psi} i \partial \Psi = \frac{1}{8}\kappa^2 \eta^{\alpha\beta} \eta^{\rho\sigma} i \partial_{2ij} \times \bar{\Psi}_i \Psi_j h_{\alpha\beta} h_{\rho\sigma} \equiv U_{1ij}^{\alpha\beta\rho\sigma} \times \bar{\Psi}_i \Psi_j h_{\alpha\beta} h_{\rho\sigma} . \quad (37)$$

The ten 4-point vertex operators are given in Table 1. Note that we do not bother to symmetrize upon the identical graviton fields.

3 Graviton Propagator and Counterterms

The gravitational Lagrangian of low energy effective field theory is,

$$\mathcal{L}_{\text{Einstein}} \equiv \frac{1}{16\pi G} (R - (D-2)\Lambda) \sqrt{-g} . \quad (38)$$

The symbols G and Λ stand for Newton's constant and the cosmological constant, respectively. The unfamiliar factor of $D-2$ multiplying Λ makes the pure gravity field equations imply $R_{\mu\nu} = \Lambda g_{\mu\nu}$ in any dimension. The symbol R stands for the Ricci scalar where our metric is spacelike and our curvature convention is,

$$R \equiv g^{\mu\nu} R_{\mu\nu} \equiv g^{\mu\nu} (\Gamma_{\nu\mu,\rho}^\rho - \Gamma_{\rho\mu,\nu}^\rho + \Gamma_{\rho\sigma}^\rho \Gamma_{\nu\mu}^\sigma - \Gamma_{\nu\sigma}^\rho \Gamma_{\rho\mu}^\sigma) . \quad (39)$$

Even though gravity is not conformally invariant, it is still useful to express it in terms of the rescaled metric (16) and connection (17),

$$\begin{aligned} \mathcal{L}_{\text{Einstein}} = \frac{1}{16\pi G} \Big\{ & a^{D-2} \tilde{R} - 2(D-1)a^{D-3} \tilde{g}^{\mu\nu} (a_{,\mu\nu} - \tilde{\Gamma}_{\mu\nu}^\rho a_{,\rho}) \\ & - (D-4)(D-1)a^{D-4} \tilde{g}^{\mu\nu} a_{,\mu} a_{,\nu} - (D-2)\Lambda a^D \Big\} \sqrt{-\tilde{g}} . \quad (40) \end{aligned}$$

The factors of a which complicate this expression are the ultimate reason there is interesting physics in this model!

We need to fix a in order to work out the graviton propagator from the Einstein Lagrangian (40). The unique, maximally symmetric solution for positive Λ is known as de Sitter space. In order to regard this as a paradigm for inflation we work on a portion of the full de Sitter manifold known as the open conformal coordinate patch. The invariant element for this is,

$$ds^2 = a^2(-d\eta^2 + d\vec{x} \cdot d\vec{x}) \quad \text{where} \quad a(\eta) = -\frac{1}{H\eta}, \quad (41)$$

and the D -dimensional Hubble constant is $H \equiv \sqrt{\Lambda/(D-1)}$. Note that the conformal time η runs from $-\infty$ to zero. For this choice of scale factor we can extract a surface term from the invariant Lagrangian and write it in the form [27],

$$\begin{aligned} \mathcal{L}_{\text{Einstein-Surface}} = & (\frac{D}{2}-1)Ha^{D-1}\sqrt{-\tilde{g}}\tilde{g}^{\rho\sigma}\tilde{g}^{\mu\nu}h_{\rho\sigma,\mu}h_{\nu 0} + a^{D-2}\sqrt{-\tilde{g}}\tilde{g}^{\alpha\beta}\tilde{g}^{\rho\sigma}\tilde{g}^{\mu\nu} \\ & \times \left\{ \frac{1}{2}h_{\alpha\rho,\beta}h_{\sigma\mu,\nu} - \frac{1}{2}h_{\alpha\beta,\rho}h_{\sigma\mu,\nu} + \frac{1}{4}h_{\alpha\beta,\rho}h_{\mu\nu,\sigma} - \frac{1}{4}h_{\alpha\rho,\mu}h_{\beta\sigma,\nu} \right\}. \end{aligned} \quad (42)$$

Gauge fixing is accomplished as usual by adding a gauge fixing term directly to the Lagrangian. One can add such a gauge fixing term and then use the well-known formalism of Allen and Jacobson [35] to solve for a fully de Sitter invariant propagator [36, 26, 37, 38, 39]. However, a curious thing happens when one uses the imaginary part of any such propagator to infer what ought to be the retarded Green's function of classical general relativity on a de Sitter background. The resulting Green's function gives a divergent response for a point mass which also fails to obey the linearized invariant Einstein equation [26]! We stress that the various propagators really do solve the gauge-fixed, linearized equations with a point source. It is the physics which is wrong, not the math. Furthermore, quantum corrections bring new problems when using de Sitter invariant gauges. The one loop scalar self-mass-squared has recently been computed in two different gauges for scalar quantum electrodynamics [44]. When the computation was done in the de Sitter invariant analogue of Feynman gauge the result was on-shell singularities! Off shell one-particle-irreducible functions need not agree in different gauges [55] but they should agree on shell [56].

The nature of the problem is the apparent inconsistency between de Sitter invariance and the manifold's linearization instability. Any propagator gives

the response (with a certain boundary condition) to a single point source. If the propagator is also de Sitter invariant then this response must be valid throughout the full de Sitter manifold. But the linearization instability precludes solving the invariant field equations for a single point source on the full manifold! This feature of the invariant theory is lost when a de Sitter invariant gauge fixing term is simply added to the action³. To understand this subtle issue, we digress to explain the difference between an “exact gauge” and an “average gauge”. The former is obtained by choosing the gauge parameter to make the fields obey some equation at each point in space and time. The other more common type of gauge fixing which we have already mentioned in the preceding paragraph is accomplished by adding some terms to the invariant Lagrangian. The functional integral representation for this type of gauge condition can be viewed as a weighted “average” of exact gauge. One can usually find the equivalence between the exact gauge conditions of the canonical formalism and the average gauge condition employed in the functional formalism. The surprising thing is that the Faddeev-Popov ansatz proven by the Sidney Coleman [57] for the flat space on the manifold R^4 is not generally valid for the gauge theory which possesses a linearization instability [58].

Because of this topological obstacle to adding covariant, average gauge fixing terms on gravity in de Sitter, except for employing the exact gauge condition on an open manifold [59] one can also avoid it either by working on the full manifold with a noncovariant gauge condition that preserves the elliptic character of the constraint equations; or else by employing a covariant, but not de Sitter invariant gauge on an open submanifold [27]. We choose the later course and employ the following analogue of the de Donder gauge fixing term of flat space,

$$\mathcal{L}_{GF} = -\frac{1}{2}a^{D-2}\eta^{\mu\nu}F_\mu F_\nu, \quad F_\mu \equiv \eta^{\rho\sigma}\left(h_{\mu\rho,\sigma} - \frac{1}{2}h_{\rho\sigma,\mu} + (D-2)Hah_{\mu\rho}\delta_\sigma^0\right). \quad (43)$$

Because our gauge condition breaks de Sitter invariance it will be necessary to contemplate noninvariant counterterms. It is therefore appropriate to digress at this point with a description of the various de Sitter symmetries and their effect upon (43). In our D -dimensional conformal coordinate system the $\frac{1}{2}D(D+1)$ de Sitter transformations take the following form:

³It results in changing the constrained equation from being elliptical to hyperbolic, which also alters the causal structure of the de Sitter geometry.

1. Spatial translations — comprising $(D-1)$ transformations.

$$\eta' = \eta , \quad (44)$$

$$x'^i = x^i + \epsilon^i . \quad (45)$$

2. Rotations — comprising $\frac{1}{2}(D-1)(D-2)$ transformations.

$$\eta' = \eta , \quad (46)$$

$$x'^i = R^{ij} x^j . \quad (47)$$

3. Dilatation — comprising 1 transformation.

$$\eta' = k \eta , \quad (48)$$

$$x'^i = k x^i . \quad (49)$$

4. Spatial special conformal transformations — comprising $(D-1)$ transformations.

$$\eta' = \frac{\eta}{1 - 2\vec{\theta} \cdot \vec{x} + \|\vec{\theta}\|^2 x \cdot x} , \quad (50)$$

$$x'^i = \frac{x^i - \theta^i x \cdot x}{1 - 2\vec{\theta} \cdot \vec{x} + \|\vec{\theta}\|^2 x \cdot x} . \quad (51)$$

It is easy to check that our gauge condition respects all of these but the spatial special conformal transformations. We will see that the other symmetries impose important restrictions upon the BPHZ counterterms which are allowed.

It is now time to solve for the graviton propagator. Because its space and time components are treated differently in our coordinate system and gauge it is useful to have an expression for the purely spatial parts of the Lorentz metric and the Kronecker delta,

$$\bar{\eta}_{\mu\nu} \equiv \eta_{\mu\nu} + \delta_\mu^0 \delta_\nu^0 \quad \text{and} \quad \bar{\delta}_\nu^\mu \equiv \delta_\nu^\mu - \delta_0^\mu \delta_\nu^0 . \quad (52)$$

The quadratic part of $\mathcal{L}_{\text{Einstein}} + \mathcal{L}_{GF}$ can be partially integrated to take the form $\frac{1}{2} h^{\mu\nu} D_{\mu\nu}{}^{\rho\sigma} h_{\rho\sigma}$, where the kinetic operator is,

$$\begin{aligned} D_{\mu\nu}{}^{\rho\sigma} \equiv & \left\{ \frac{1}{2} \bar{\delta}_\mu^{(\rho} \bar{\delta}_\nu^{\sigma)} - \frac{1}{4} \eta_{\mu\nu} \eta^{\rho\sigma} - \frac{1}{2(D-3)} \delta_\mu^0 \delta_\nu^0 \delta_0^\rho \delta_0^\sigma \right\} D_A \\ & + \delta_{(\mu}^0 \bar{\delta}_{\nu)}^{(\rho} \delta_0^{\sigma)} D_B + \frac{1}{2} \left(\frac{D-2}{D-3} \right) \delta_\mu^0 \delta_\nu^0 \delta_0^\rho \delta_0^\sigma D_C , \end{aligned} \quad (53)$$

and the three scalar differential operators are,

$$D_A \equiv \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu) , \quad (54)$$

$$D_B \equiv \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu) - \frac{1}{D} \left(\frac{D-2}{D-1} \right) R \sqrt{-g} , \quad (55)$$

$$D_C \equiv \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu) - \frac{2}{D} \left(\frac{D-3}{D-1} \right) R \sqrt{-g} . \quad (56)$$

The graviton propagator in this gauge takes the form of a sum of constant index factors times scalar propagators,

$$i[\mu\nu\Delta_{\rho\sigma}](x; x') = \sum_{I=A,B,C} [\mu\nu T_{\rho\sigma}^I] i\Delta_I(x; x') . \quad (57)$$

The three scalar propagators invert the various scalar kinetic operators,

$$D_I \times i\Delta_I(x; x') = i\delta^D(x - x') \quad \text{for} \quad I = A, B, C , \quad (58)$$

and we will presently give explicit expressions for them. The index factors are,

$$[\mu\nu T_{\rho\sigma}^A] = 2\bar{\eta}_{\mu(\rho}\bar{\eta}_{\sigma)\nu} - \frac{2}{D-3}\bar{\eta}_{\mu\nu}\bar{\eta}_{\rho\sigma} , \quad (59)$$

$$[\mu\nu T_{\rho\sigma}^B] = -4\delta_{(\mu}^0\bar{\eta}_{\nu)(\rho}\delta_{\sigma)}^0 , \quad (60)$$

$$[\mu\nu T_{\rho\sigma}^C] = \frac{2}{(D-2)(D-3)} [(D-3)\delta_\mu^0\delta_\nu^0 + \bar{\eta}_{\mu\nu}] [(D-3)\delta_\rho^0\delta_\sigma^0 + \bar{\eta}_{\rho\sigma}] . \quad (61)$$

With these definitions and equation (58) for the scalar propagators it is straightforward to verify that the graviton propagator (57) indeed inverts the gauge-fixed kinetic operator,

$$D_{\mu\nu}{}^{\rho\sigma} \times i[\rho\sigma\Delta^{\alpha\beta}](x; x') = \delta_\mu^{(\alpha}\delta_\nu^{\beta)} i\delta^D(x - x') . \quad (62)$$

The scalar propagators can be expressed in terms of the following function of the invariant length $\ell(x; x')$ between x^μ and x'^μ ,

$$y(x; x') \equiv 4\sin^2\left(\frac{1}{2}H\ell(x; x')\right) = aa'H^2\Delta x^2(x; x') , \quad (63)$$

$$= aa'H^2\left(\|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\delta)^2\right) . \quad (64)$$

The most singular term for each case is the propagator for a massless, conformally coupled scalar [60],

$$i\Delta_{\text{cf}}(x; x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \Gamma\left(\frac{D}{2}-1\right) \left(\frac{4}{y}\right)^{\frac{D}{2}-1}. \quad (65)$$

The A -type propagator obeys the same equation as that of a massless, minimally coupled scalar. It has long been known that no de Sitter invariant solution exists [61]. If one elects to break de Sitter invariance while preserving homogeneity (44-45) and isotropy (46-47) — this is known as the “E(3)” vacuum [62] — the minimal solution is [45, 46],

$$\begin{aligned} i\Delta_A(x; x') &= i\Delta_{\text{cf}}(x; x') \\ &+ \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left\{ \frac{D}{D-4} \frac{\Gamma^2(\frac{D}{2})}{\Gamma(D-1)} \left(\frac{4}{y}\right)^{\frac{D}{2}-2} \pi \cot\left(\frac{\pi}{2}D\right) + \ln(aa') \right\} \\ &+ \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \sum_{n=1}^{\infty} \left\{ \frac{1}{n} \frac{\Gamma(n+D-1)}{\Gamma(n+\frac{D}{2})} \left(\frac{y}{4}\right)^n - \frac{1}{n-\frac{D}{2}+2} \frac{\Gamma(n+\frac{D}{2}+1)}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right\}. \end{aligned} \quad (66)$$

Note that this solution breaks dilatation invariance (48-49) in addition to the spatial special conformal invariance (50-51) broken by the gauge condition. By convoluting naive de Sitter transformations with the compensating diffeomorphisms necessary to restore our gauge condition (43) one can show that the breaking of dilatation invariance is physical whereas the apparent breaking of spatial special conformal invariance is a gauge artifact [63].

The B -type and C -type propagators possess de Sitter invariant (and also unique) solutions,

$$i\Delta_B(x; x') = i\Delta_{\text{cf}}(x; x') - \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \sum_{n=0}^{\infty} \left\{ \frac{\Gamma(n+D-2)}{\Gamma(n+\frac{D}{2})} \left(\frac{y}{4}\right)^n - \frac{\Gamma(n+\frac{D}{2})}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right\}, \quad (67)$$

$$\begin{aligned} i\Delta_C(x; x') &= i\Delta_{\text{cf}}(x; x') + \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \sum_{n=0}^{\infty} \left\{ (n+1) \frac{\Gamma(n+D-3)}{\Gamma(n+\frac{D}{2})} \left(\frac{y}{4}\right)^n \right. \\ &\quad \left. - \left(n-\frac{D}{2}+3\right) \frac{\Gamma(n+\frac{D}{2}-1)}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right\}. \end{aligned} \quad (68)$$

These expressions might seem daunting but they are actually simple to use because the infinite sums vanish in $D = 4$, and each term in these sums

goes like a positive power of $y(x; x')$. This means the infinite sums can only contribute when multiplied by a divergent term, and even then only a small number of terms can contribute. Note also that the B -type and C -type propagators agree with the conformal propagator in $D = 4$.

In view of the subtle problems associated with the graviton propagator in what seemed to be perfectly valid, de Sitter invariant gauges [26, 27], it is well to review the extensive checks that have been made on the consistency of this noninvariant propagator. This propagator has been shown to obey the Ward identity at tree order [65] and one loop [72]. In addition, the only fully dimensionally regulated loop results for quantum gravity on de Sitter background have been obtained using it [1, 43, 66, 67] as well.

It remains to deal with the local counterterms we must add, order-by-order in perturbation theory, to absorb divergences in the sense of BPHZ renormalization. The particular counterterms which renormalize the fermion self-energy must obviously involve a single $\bar{\psi}$ and a single ψ . At one loop order the superficial degree of divergence of quantum gravitational contributions to the fermion self-energy is three, so the necessary counterterms can involve zero, one, two or three derivatives. These derivatives can either act upon the fermi fields or upon the metric, in which case they must be organized into curvatures or derivatives of curvatures. We will first exhaust the possible invariant counterterms for a general renormalized fermion mass and a general background geometry, and then specialize to the case of the lowest order of mass term in de Sitter background. We close with a discussion of possible noninvariant counterterms.

All one loop corrections from quantum gravity must carry a factor of $\kappa^2 \sim \text{mass}^{-2}$. There will be additional dimensions associated with derivatives and with the various fields, and the balance must be struck using the renormalized fermion mass, m . Hence the only invariant counterterm with no derivatives has the form,

$$\kappa^2 m^3 \bar{\psi} \psi \sqrt{-g} . \quad (69)$$

With one derivative we can always partially integrate to act upon the ψ field, so the only invariant counterterm is,

$$\kappa^2 m^2 \bar{\psi} i \not{D} \psi \sqrt{-g} . \quad (70)$$

Two derivatives can either act upon the fermions or else on the metric to produce curvatures. We can organize the various possibilities as follows,

$$\kappa^2 m \bar{\psi} (i \not{D})^2 \psi \sqrt{-g} \quad , \quad \kappa^2 m R \bar{\psi} \psi \sqrt{-g} . \quad (71)$$

Three derivatives can be all acted on the fermions, or one on the fermions and two in the form of curvatures, or there can be a differentiated curvature,

$$\begin{aligned} & \kappa^2 \bar{\psi} \left((i\mathcal{D})^2 + \frac{R}{D(D-1)} \right) i\mathcal{D}\psi \sqrt{-g} \quad , \quad \kappa^2 R \bar{\psi} i\mathcal{D}\psi \sqrt{-g} \quad , \\ & \kappa^2 e_{\mu m} \left(R^{\mu\nu} - \frac{1}{D} g^{\mu\nu} R \right) \bar{\psi} \gamma^m i\mathcal{D}_\nu \psi \sqrt{-g} \quad , \quad \kappa^2 e^\mu_m R_{,\mu} \bar{\psi} \gamma^m \psi \sqrt{-g} \quad . \end{aligned} \quad (72)$$

The counterterms with zero mass (72) have already been discussed in [1]. Because we are dealing with the order m contribution, we do not consider counterterms (69) and (70) and only focus on (71) for the purpose of the current computation. The specialization of the invariant counter-Lagrangian we require to de Sitter background at order m is therefore,

$$\Delta\mathcal{L}_{\text{inv}} = \lambda_1 \kappa^2 m \bar{\psi} (i\mathcal{D})^2 \psi \sqrt{-g} + \lambda_2 \kappa^2 m R \bar{\psi} \psi \sqrt{-g} \quad , \quad (73)$$

$$\longrightarrow \lambda_1 \kappa^2 \bar{\Psi} m (i\partial a^{-1} i\partial) \Psi + \lambda_2 \kappa^2 (D-1) D H^2 m a \bar{\Psi} \Psi \quad . \quad (74)$$

Here λ_1 and λ_2 are D -dependent constants which are dimensionless. The associated vertex operators are,

$$C_{1ij} \equiv \lambda_1 \kappa^2 \left(\frac{m}{a} \partial^2 + m H \gamma^0 \partial \right) = \lambda_1 \kappa^2 m \partial a^{-1} \partial \quad , \quad (75)$$

$$C_{2ij} \equiv \lambda_2 D(D-1) \kappa^2 H^2 m a \quad . \quad (76)$$

Of course C_1 is the higher derivative counterterm mentioned in section 1. It will renormalize the most singular terms — coming from the $i\Delta_{\text{cf}}$ part of the graviton propagator — which are unimportant because they are suppressed by powers of the scale factor. The other vertex operator, C_2 , is a sort of dimensionful field strength renormalization in de Sitter background. It will renormalize the less singular contributions which derive physically from inflationary particle production.

We do not use the background field technique in background field gauge [75, 76, 77, 78]. We must therefore come to terms with the possibility that divergences may arise which require noninvariant counterterms. What form can these counterterms take? Applying the BPHZ theorem [6, 7, 8, 9] to the gauge-fixed theory in de Sitter background implies that the relevant counterterms must still consist of κ^2 times a spinor differential operator with the dimension of mass-cubed, involving no more than three derivatives and acting between $\bar{\Psi}$ and Ψ . As the only dimensionful constant in our problem,

powers of H must be used to make up whatever dimensions are not supplied by derivatives.

Because dimensional regularization respects diffeomorphism invariance, it is only the gauge fixing term (43) that permits noninvariant counterterms.⁴ Conversely, noninvariant counterterms must respect the residual symmetries of the gauge condition. Homogeneity (44-45) implies that the spinor differential operator cannot depend upon the spatial coordinate x^i . Similarly, isotropy (46-47) requires that any spatial derivative operators ∂_i must either be contracted into γ^i or another spatial derivative. Owing to the identity,

$$(\gamma^i \partial_i)^2 = -\nabla^2, \quad (77)$$

we can think of all spatial derivatives as contracted into γ^i . Although the temporal derivative is not required to be multiplied by γ^0 we lose nothing by doing so provided additional dependence upon γ^0 is allowed.

The final residual symmetry is dilatation invariance (48-49). It has the crucial consequence that derivative operators can only appear in the form $a^{-1} \partial_\mu$. In addition the entire counterterm must have an overall factor of a , and there can be no other dependence upon η . So the most general order m counterterm consistent with our gauge condition takes the form,

$$\Delta \mathcal{L}_{\text{non}} = \kappa^2 H^2 m a \bar{\Psi} \mathcal{S} \left((Ha)^{-1} \gamma^0 \partial_0, (Ha)^{-1} \gamma^i \partial_i \right) \Psi, \quad (78)$$

where the spinor function $\mathcal{S}(b, c)$ is at most a second order polynomial function of its arguments, and it may involve γ^0 in an arbitrary way.

Three more principles constrain the order m noninvariant counterterms. The first of these principles is that the fermion self-energy at order m involves only even powers of gamma matrices. This follows because the three-point vertices, the four-point vertices and the fermion propagator all consist of an even number of γ 's at order m^1 and an odd number of γ 's at order m^0 . The diagram which consists of one 4-point vertex possesses an even number of

⁴One might think that the they could come as well from the fact that the vacuum breaks de Sitter invariance, but symmetries broken by the vacuum do not introduce new counterterms [79]. Highly relevant, explicit examples are provided by recent computations for a massless, minimally coupled scalar with a quartic self-interaction in the same locally de Sitter background used here. The vacuum in this theory also breaks de Sitter invariance but noninvariant counterterms fail to arise even at *two loop* order in either the expectation value of the stress tensor [45, 46] or the self-mass-squared [47]. It is also relevant that the one loop vacuum polarization from (massless, minimally coupled) scalar quantum electrodynamics is free of noninvariant counterterms in the same background [80].

gamma matrices at order m . The contribution from any diagram with two 3-point vertices consists of three factors involving gamma matrices: one factor from the fermion propagator and one factor from each of the two vertices. At order m such a product consists of one even and two odd factors, so it contains an even number of gamma matrices. This principle fixes the dependence upon γ^0 and allows us to express the spinor differential operator in terms of just ten constants β_i ,

$$\begin{aligned} & \kappa^2 H^2 m a \mathcal{S}((Ha)^{-1} \gamma^0 \partial_0, (Ha)^{-1} \gamma^i \partial_i) \\ &= \kappa^2 m a \left\{ \beta_1 (a^{-1} \gamma^0 \partial_0)^2 + \beta_2 [(a^{-1} \gamma^0 \partial_0)(a^{-1} \gamma^i \partial_i)] + \beta_3 (a^{-1} \gamma^i \partial_i)^2 \right. \\ & \quad \left. + H \gamma^0 (\beta_4 (a^{-1} \gamma^0 \partial_0) + \beta_5 (a^{-1} \gamma^i \partial_i)) + H^2 \beta_6 \right\}. \end{aligned} \quad (79)$$

In this expansion, but for the rest of this section only, we define noncommuting factors within square brackets to be symmetrically ordered, for example,

$$[(a^{-1} \gamma^0 \partial_0)(a^{-1} \gamma^i \partial_i)] \equiv \frac{1}{2} (a^{-1} \gamma^0 \partial_0)(a^{-1} \gamma^i \partial_i) + \frac{1}{2} (a^{-1} \gamma^i \partial_i)(a^{-1} \gamma^0 \partial_0). \quad (80)$$

The second principle is that our gauge condition (43) becomes Poincaré invariant in the flat space limit of $H \rightarrow 0$, where the conformal time is $\eta = -e^{-Ht}/H$ with t held fixed. In that limit only the three quadratic terms of (79) survive,

$$\begin{aligned} & \lim_{H \rightarrow 0} \kappa^2 H^2 m a \mathcal{S}((Ha)^{-1} \gamma^0 \partial_0, (Ha)^{-1} \gamma^i \partial_i) \\ &= \kappa^2 m a \left\{ \beta_1 (a^{-1} \gamma^0 \partial_0)^2 + \beta_2 [(a^{-1} \gamma^0 \partial_0)(a^{-1} \gamma^i \partial_i)] + \beta_3 (a^{-1} \gamma^i \partial_i)^2 \right\} \end{aligned} \quad (81)$$

Because the entire theory is Poincaré invariant in that limit, these three terms must sum to a term proportional to $(\gamma^\mu \partial_\mu)^2$, which implies,

$$\beta_1 = \frac{1}{2} \beta_2 = \beta_3. \quad (82)$$

But in that case the three quadratic terms sum to give (75),

$$\kappa^2 m a \left\{ (a^{-1} \gamma^0 \partial_0)^2 + 2[(a^{-1} \gamma^0 \partial_0)(a^{-1} \gamma^i \partial_i)] + (a^{-1} \gamma^i \partial_i)^2 \right\} = \kappa^2 m \not{\partial} a^{-1} \not{\partial}. \quad (83)$$

Because it is the same as one of the invariant counterterms, it need not be included in \mathcal{S} . Besides, the final term in (79) recovers the other invariant counterterm (76). So the two remaining noninvariant counterterms we need to consider in (79) are,

$$\Delta\mathcal{L}_{non} = \overline{\Psi} \left\{ \beta_4(a^{-1}\gamma^0\partial_0) + \beta_5(a^{-1}\gamma^i\partial_i) \right\} \Psi . \quad (84)$$

However, these two terms are not independent of the last term in (75). Therefore we could chose any four independent counterterm operators we need for this computation,

$$\alpha_1\kappa^2\frac{m}{a}\partial^2 \quad , \quad \alpha_4\kappa^2H^2ma \quad , \quad (85)$$

$$\alpha_2\kappa^2mH\partial_0 \quad , \quad \alpha_3\kappa^2mH\gamma^0\overline{\partial} . \quad (86)$$

4 Contributions from the 4-Point Vertices

In this section we evaluate the contributions from 4-point vertex operators of Table 1. The generic diagram topology is depicted in Fig. 1. The analytic form is,

$$-i\left[{}_i\Sigma_j^{4\text{pt}}\right](x;x') = \sum_{I=1}^{10} iU_{Iij}^{\alpha\beta\rho\sigma} i\left[{}_{\alpha\beta}\Delta_{\rho\sigma}\right](x;x') \delta^D(x-x') . \quad (87)$$

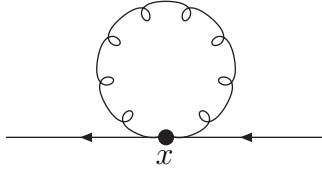


Fig. 1: Contribution from 4-point vertices.

And the generic contraction for each of the vertex operators in Table 1 is given in Table 2.

From an examination of the generic contractions in Table 2 it is apparent that we must work out how the three index factors $[\alpha_\beta T^I_{\rho\sigma}]$ which make up the graviton propagator contract into $\eta^{\alpha\beta}$ and $\eta^{\alpha\rho}$. For the *A*-type and *B*-type

I	$i[\alpha\beta\Delta_{\rho\sigma}](x; x') iU_I^{\alpha\beta\rho\sigma} \delta^D(x-x')$
1	$-\frac{1}{8}\kappa^2 i[\alpha_\rho\Delta^\rho_\sigma](x; x) \not{\partial} \delta^D(x-x')$
2	$\frac{1}{4}\kappa^2 i[\alpha^\beta\Delta_{\alpha\beta}](x; x) \not{\partial} \delta^D(x-x')$
3	$\frac{1}{4}\kappa^2 i[\alpha_\rho\Delta_{\rho\sigma}](x; x) \gamma^\rho \partial^\sigma \delta^D(x-x')$
4	$-\frac{3}{8}\kappa^2 i[\alpha_\beta\Delta_{\alpha\sigma}](x; x) \gamma^\beta \partial^\sigma \delta^D(x-x')$
5	$-\frac{i}{4}\kappa^2 \partial'_\mu i[\alpha_\rho\Delta_{\rho\sigma}](x; x') \gamma^\rho J^{\sigma\mu} \delta^D(x-x')$
6	$\frac{i}{8}\kappa^2 \partial'_\mu i[\alpha_\beta\Delta_{\alpha\sigma}](x; x') \gamma^\mu J^{\beta\sigma} \delta^D(x-x')$
7	$\frac{i}{4}\kappa^2 \partial_\mu i[\alpha_\beta\Delta_{\alpha\sigma}](x; x) \gamma^\beta J^{\sigma\mu} \delta^D(x-x')$
8	$\frac{i}{4}\kappa^2 \partial'^\beta i[\alpha_\beta\Delta_{\rho\sigma}](x; x') \gamma^\rho J^{\sigma\alpha} \delta^D(x-x')$
9	$-\frac{i}{8}\kappa^2 am i[\alpha_\rho\Delta^\rho_\sigma](x; x) \delta^D(x-x')$
10	$\frac{i}{4}\kappa^2 am i[\alpha^\beta\Delta_{\alpha\beta}](x; x) \delta^D(x-x')$

Table 2: Generic 4-point contractions

index factors the various contractions give,

$$\eta^{\alpha\beta} [\alpha_\beta T_{\rho\sigma}^A] = -\left(\frac{4}{D-3}\right) \bar{\eta}_{\rho\sigma} \quad , \quad \eta^{\alpha\rho} [\alpha_\beta T_{\rho\sigma}^A] = \left(D - \frac{2}{D-3}\right) \bar{\eta}_{\beta\sigma} \quad , \quad (88)$$

$$\eta^{\alpha\beta} [\alpha_\beta T_{\rho\sigma}^B] = 0 \quad , \quad \eta^{\alpha\rho} [\alpha_\beta T_{\rho\sigma}^B] = -(D-1) \delta_\beta^0 \delta_\sigma^0 + \bar{\eta}_{\beta\sigma} \quad , \quad (89)$$

For the C -type index factor they are,

$$\begin{aligned} \eta^{\alpha\beta} [\alpha_\beta T_{\rho\sigma}^C] &= \left(\frac{4}{D-2}\right) \delta_\rho^0 \delta_\sigma^0 + \frac{4}{(D-2)(D-3)} \bar{\eta}_{\rho\sigma} \quad , \\ \eta^{\alpha\rho} [\alpha_\beta T_{\rho\sigma}^C] &= -2\left(\frac{D-3}{D-2}\right) \delta_\beta^0 \delta_\sigma^0 + \frac{2}{(D-2)(D-3)} \bar{\eta}_{\beta\sigma} \quad . \end{aligned} \quad (90)$$

At order m we actually only require double contractions. For the A -type index factor these are,

$$\begin{aligned} \eta^{\alpha\beta} \eta^{\rho\sigma} [\alpha_\beta T_{\rho\sigma}^A] &= -4\left(\frac{D-1}{D-3}\right) \quad , \\ \eta^{\alpha\rho} \eta^{\beta\sigma} [\alpha_\beta T_{\rho\sigma}^A] &= D(D-1) - 2\left(\frac{D-1}{D-3}\right) \quad . \end{aligned} \quad (91)$$

The double contractions of the B -type and C -type index factors are,

$$\eta^{\alpha\beta} \eta^{\rho\sigma} [\alpha_\beta T_{\rho\sigma}^B] = 0 \quad , \quad \eta^{\alpha\rho} \eta^{\beta\sigma} [\alpha_\beta T_{\rho\sigma}^B] = 2(D-1) \quad , \quad (92)$$

I	J	$i[\alpha\beta T_{\rho\sigma}^J] i\Delta_J(x; x') iU_I^{\alpha\beta\rho\sigma} \delta^D(x-x')$
9	A	$\frac{i}{2}\kappa^2 \frac{(D-1)}{(D-3)} am i\Delta_A(x; x) \delta^D(x-x')$
9	C	$-\frac{i}{(D-2)(D-3)}\kappa^2 am i\Delta_C(x; x) \delta^D(x-x')$
10	A	$\frac{i}{4}\kappa^2 [D(D-1) - 2\frac{(D-1)}{(D-3)}] am i\Delta_A(x; x) \delta^D(x-x')$
10	B	$\frac{i}{2}\kappa^2 (D-1) am i\Delta_B(x; x) \delta^D(x-x')$
10	C	$\frac{i}{2}\kappa^2 \frac{(D^2-5D+8)}{(D-2)(D-3)} am i\Delta_C(x; x) \delta^D(x-x')$

Table 3: 4-point contribution from each part of the graviton propagator at order m . The vertices 1-8 could only give the contribution at order m^0 .

$$\eta^{\alpha\beta}\eta^{\rho\sigma} [\alpha\beta T_{\rho\sigma}^C] = \frac{8}{(D-2)(D-3)} \quad , \quad \eta^{\alpha\rho}\eta^{\beta\sigma} [\alpha\beta T_{\rho\sigma}^C] = 2\frac{(D^2-5D+8)}{(D-2)(D-3)} . \quad (93)$$

Table 3 was generated from Table 2 by expanding the graviton propagator in terms of index factors,

$$i[\alpha\beta\Delta_{\rho\sigma}](x; x') = [\alpha\beta T_{\rho\sigma}^A] i\Delta_A(x; x') + [\alpha\beta T_{\rho\sigma}^B] i\Delta_B(x; x') + [\alpha\beta T_{\rho\sigma}^C] i\Delta_C(x; x') . \quad (94)$$

We then perform the relevant contractions using the previous identities.

From Table 3 it is apparent that we require the coincidence limits on each of the scalar propagators. For the A -type propagator these are,

$$\lim_{x' \rightarrow x} i\Delta_A(x; x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left\{ -\pi \cot\left(\frac{\pi}{2}D\right) + 2\ln(a) \right\} . \quad (95)$$

The analogous coincidence limits for the B -type propagator are actually finite in $D = 4$ dimensions,

$$\lim_{x' \rightarrow x} i\Delta_B(x; x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times -\frac{1}{D-2} . \quad (96)$$

The same is true for the coincidence limits of the C -type propagator,

$$\lim_{x' \rightarrow x} i\Delta_C(x; x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \times \frac{1}{(D-2)(D-3)} . \quad (97)$$

I	J	$i am \delta^D(x-x')$
9	A	$-\frac{(D-1)}{(D-3)} A$
9	B	0
9	C	$-\frac{1}{(D-2)^2(D-3)^2}$
10	A	$-\frac{1}{2}[D(D-1) - \frac{2(D-1)}{(D-3)}] A$
10	B	$-\frac{1}{2} \frac{(D-1)}{(D-2)}$
10	C	$\frac{1}{2} \frac{(D^2-5D+8)}{(D-2)^2(D-3)^2}$

Table 4: The 4-point contributions at order m. All contributions are multiplied by $\frac{\kappa^2 H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}$. Here $A \equiv \frac{\pi}{2} \cot(\frac{D\pi}{2}) - \ln(a)$.

We apply the various coincidence limits to each contraction in Table 3 and present the order m, 4-point contributions in Table 4. The total summation for this local contributions is quite simple,

$$\begin{aligned}
-i[\Sigma^{4\text{pt}}](x; x') &= \frac{i\kappa^2 H^2 m a}{2^{D+1} \pi^{\frac{D}{2}}} \frac{H^{D-4}}{(D-4)} \frac{-\Gamma(D+1)}{\Gamma(\frac{D}{2})} \delta^D(x-x') \\
&\quad + \frac{i\kappa^2 H^2}{16\pi^2} m a [12 \ln a - 1] \delta^4(x-x') . \quad (98)
\end{aligned}$$

5 Contributions from the 3-Point Vertices

In this section we evaluate the contributions from two 3-point vertex operators. The generic diagram topology is depicted in Fig. 2. The analytic form is,

$$-i[\Sigma_j^{3\text{pt}}](x; x') = \sum_{I=1}^4 iV_{Ik}^{\alpha\beta}(x) i[kS_\ell](x; x') \sum_{J=1}^4 iV_{J\ell}^{\rho\sigma}(x') i[\alpha_\beta \Delta_{\rho\sigma}](x; x') . \quad (99)$$

I	J	$iV_I^{\alpha\beta}(x) i[S](x; x') iV_J^{\rho\sigma}(x') i[\alpha_\beta \Delta_{\rho\sigma}](x; x')$
1	1	$-\frac{1}{4}\kappa^2 \partial'_\mu \{ \not{\partial} i[S](x; x') \gamma^\mu i[\alpha_\beta \Delta^\rho_\rho](x; x') \}$
1	2	$\frac{1}{4}\kappa^2 \partial'^\rho \{ \not{\partial} i[S](x; x') \gamma^\sigma i[\alpha_\beta \Delta_{\rho\sigma}](x; x') \}$
1	3	$\frac{1}{4}i\kappa^2 \not{\partial} i[S](x; x') \gamma^\rho J^{\sigma\mu} \partial'_\mu i[\alpha_\beta \Delta_{\rho\sigma}](x; x')$
1	4	$\frac{1}{4}i\kappa^2 am \not{\partial} i[S](x; x') i[\alpha_\beta \Delta^\rho_\rho](x; x')$
2	1	$\frac{1}{4}\kappa^2 \partial'_\mu \{ \gamma^\alpha \partial^\beta i[S](x; x') \gamma^\mu i[\alpha_\beta \Delta^\rho_\rho](x; x') \}$
2	2	$-\frac{1}{4}\kappa^2 \partial'^\rho \{ \gamma^\alpha \partial^\beta i[S](x; x') \gamma^\sigma i[\alpha_\beta \Delta_{\rho\sigma}](x; x') \}$
2	3	$-\frac{1}{4}i\kappa^2 \gamma^\alpha \partial^\beta i[S](x; x') \gamma^\rho J^{\sigma\mu} \partial'_\mu i[\alpha_\beta \Delta_{\rho\sigma}](x; x')$
2	4	$-\frac{1}{4}i\kappa^2 am \gamma^\alpha \partial^\beta i[S](x; x') i[\alpha_\beta \Delta^\rho_\rho](x; x')$
3	1	$-\frac{1}{4}i\kappa^2 \partial'_\nu \{ \gamma^\alpha J^{\beta\mu} i[S](x; x') \gamma^\nu \partial_\mu i[\alpha_\beta \Delta^\rho_\rho](x; x') \}$
3	2	$\frac{1}{4}i\kappa^2 \partial'^\rho \{ \gamma^\alpha J^{\beta\mu} i[S](x; x') \gamma^\sigma \partial_\mu i[\alpha_\beta \Delta_{\rho\sigma}](x; x') \}$
3	3	$-\frac{1}{4}\kappa^2 \gamma^\alpha J^{\beta\mu} i[S](x; x') \gamma^\rho J^{\sigma\nu} \partial_\mu \partial'_\nu i[\alpha_\beta \Delta_{\rho\sigma}](x; x')$
3	4	$-\frac{1}{4}\kappa^2 am \gamma^\alpha J^{\beta\mu} i[S](x; x') \partial_\mu i[\alpha_\beta \Delta^\rho_\rho](x; x')$
4	1	$-\frac{1}{4}i\kappa^2 \partial'_\mu \{ am i[S](x; x') \gamma^\mu i[\alpha_\beta \Delta^\rho_\rho](x; x') \}$
4	2	$\frac{1}{4}i\kappa^2 \partial'^\rho \{ am i[S](x; x') \gamma^\sigma i[\alpha_\beta \Delta_{\rho\sigma}](x; x') \}$
4	3	$-\frac{1}{4}\kappa^2 am i[S](x; x') \gamma^\rho J^{\sigma\mu} \partial_\mu i[\alpha_\beta \Delta_{\rho\sigma}](x; x')$
4	4	$-\frac{1}{4}\kappa^2 a^2 m^2 i[S](x; x') i[\alpha_\beta \Delta^\rho_\rho](x; x')$

Table 5: Generic Contributions from the 3-Point Vertices.

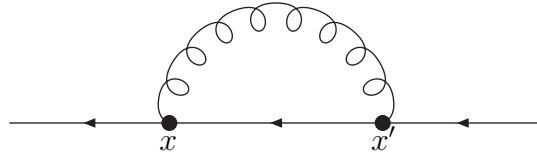


Fig. 2: Contribution from two 3-point vertices.

Because there are four 3-point vertex operators in (36), there are sixteen vertex products in (99). We label each contribution by the numbers on its vertex pair, for example,

$$[I-J] \equiv iV_I^{\alpha\beta}(x) \times i[S](x; x') \times iV_J^{\rho\sigma}(x') \times i[\alpha_\beta \Delta_{\rho\sigma}](x; x') . \quad (100)$$

Table 5 gives the generic reductions, before decomposing the graviton propagator⁵. Most of these reductions are straightforward but one subtlety deserve mention, that is, derivatives on external lines must be partially integrated back on the entire diagram. This happens whenever the second vertex is $J=1$ or $J=2$, for example,

$$[2-2] \equiv -\frac{i\kappa}{2}\gamma^\alpha i\partial^\beta \times i[S](x; x') \times -\frac{i\kappa}{2}\gamma^\rho i\partial'_{\text{ext}}{}^\sigma \times i[\alpha_\beta \Delta_{\rho\sigma}](x; x') , \quad (101)$$

$$= -\frac{\kappa^2}{4}\partial'^\sigma \left\{ \gamma^\alpha \partial^\beta i[S](x; x') \gamma^\rho i[\alpha_\beta \Delta_{\rho\sigma}](x; x') \right\}. \quad (102)$$

Another simplification we might use for later contractions is that the Dirac slash of the conformal part of fermion propagator gives a delta function,

$$i\cancel{\partial}i[S]_{\text{cf}}(x; x') = i\delta^D(x - x') . \quad (103)$$

5.1 Conformal Contributions

The key to achieving a tractable reduction of the diagrams of Fig. 2 is that the first term of each of the scalar propagators $i\Delta_I(x; x')$ is the conformal propagator $i\Delta_{\text{cf}}(x; x')$. The sum of the three index factors also gives a simple tensor, so it is very efficient to write the graviton propagator in the form,

$$i[\mu\nu\Delta_{\rho\sigma}](x; x') = \left[2\eta_{\mu(\rho}\eta_{\sigma)\nu} - \frac{2}{D-2}\eta_{\mu\nu}\eta_{\rho\sigma} \right] i\Delta_{\text{cf}}(x; x') + \sum_{I=A,B,C} [\mu\nu T_{\rho\sigma}^I] i\delta\Delta_I(x; x') , \quad (104)$$

where $i\delta\Delta_I(x; x') \equiv i\Delta_I(x; x') - i\Delta_{\text{cf}}(x; x')$. In this subsection we evaluate the contribution to (99) using the 3-point vertex operators (36) and the fermion propagator (34) but only the conformal part of the graviton propagator,

$$i[\mu\nu\Delta_{\rho\sigma}](x; x') \longrightarrow \left[2\eta_{\mu(\rho}\eta_{\sigma)\nu} - \frac{2}{D-2}\eta_{\mu\nu}\eta_{\rho\sigma} \right] i\Delta_{\text{cf}}(x; x') \equiv [\alpha_\beta T_{\rho\sigma}^{\text{cf}}] i\Delta_{\text{cf}}(x; x) . \quad (105)$$

We carry out the reduction in three stages. In the first stage the conformal part (105) of the graviton propagator is substituted into the generic results

⁵We would not consider the 4-4 contraction because it is an order m^2 contribution.

from Table 5 and the contractions are performed. We also make use of gamma matrix identities such as (9) and,

$$\gamma^\rho J^{\beta\mu} + \gamma^\beta J^{\rho\mu} = \frac{i}{2}(\gamma^\rho \eta^{\beta\mu} + \gamma^\beta \eta^{\rho\mu}) - i\gamma^\mu \eta^{\rho\beta} \quad , \quad \gamma_\alpha J^{\alpha\mu} = -\frac{i}{2}(D-1)\gamma^\mu. \quad (106)$$

We do not at this stage act any derivatives on the fermion propagator. The results of these reductions are summarized in Table 6. Because the conformal tensor factor $[\alpha\beta T_{\rho\sigma}^{\text{cf}}]$ contains three distinct terms, and because the factors of $\gamma^\alpha J^{\beta\mu}$ in Table 5 can contribute different terms with a distinct structure, we have sometimes broken up the result for a given vertex pair into parts. These parts are distinguished in Table 6 and subsequently by subscripts taken from the lower case Latin letters.

In the second stage we substitute the conformal part of the graviton propagator,

$$i\Delta_{\text{cf}}(x; x') = \frac{\Gamma(\frac{D}{2}-1)}{4\pi^{\frac{D}{2}}} \frac{(aa')^{1-\frac{D}{2}}}{\Delta x^{D-2}} \quad , \quad (107)$$

and decompose the fermion propagator (34) into the conformal part, the flat spacetime mass term, $n = 0$ part and $n \geq 1$ part of the infinite series expansion⁶. In the final stage we act the derivatives. We start from the most singular contribution in Table 6, which substitutes the conformal parts of the fermion propagator into the contraction 1-4, 2-4, 3-4, 4-1, 4-2 and 4-3⁷. The contraction 1-4 and 2-4 vanish owing to the equation (103) and owing to the zero contribution from D powers of the coordinate separation in Dimensional regularization. We also must remember that $[\Sigma](x; x')$ will be used inside an integral in the quantum-corrected Dirac equation (1). For that purpose the most singular term at $x'^\mu = x^\mu$ is quadratically divergent in $D=4$ dimensions. Hence we first conveniently⁸ employ the following identities to express the rest of them as a less singular form,

$$\frac{1}{\Delta x^{2D-2}} = \frac{\partial^2}{2(D-2)^2} \frac{1}{\Delta x^{2D-4}} \quad , \quad \frac{\Delta x_\mu}{\Delta x^{2D-2}} = \frac{-\partial_\mu}{2(D-2)} \frac{1}{\Delta x^{2D-4}}. \quad (108)$$

⁶We will explain why we separate the $n = 0$ part from the rest of the infinite series expansion in a later paragraph.

⁷The contraction 4-4 is an order m^2 contribution.

⁸Some individual term is easier written as a derivative with respect to x^μ acting upon a less singular coordinate separation than taking the derivative directly.

I	J	sub	$iV_I^{\alpha\beta}(x) i[S](x; x') iV_J^{\rho\sigma}(x') [\alpha\beta T_{\rho\sigma}^{\text{cf}}] i\Delta_{\text{cf}}(x; x')$
1	1		$\frac{D}{D-2}\kappa^2\partial'_\mu\{\partial i[S](x; x')\gamma^\mu i\Delta_{\text{cf}}(x; x')\}$
1	2		$-\frac{1}{D-2}\kappa^2\partial'_\mu\{\partial i[S](x; x')\gamma^\mu i\Delta_{\text{cf}}(x; x')\}$
1	3		$-\frac{(D-1)}{2(D-2)}\kappa^2 \not{\partial} i[S](x; x') \not{\partial} i\Delta_{\text{cf}}(x; x')$
1	4		$-\frac{D}{D-2}\kappa^2 iam \not{\partial} i[S](x; x') i\Delta_{\text{cf}}(x; x')$
2	1		$-\frac{1}{D-2}\kappa^2\partial'_\mu\{\partial i[S](x; x')\gamma^\mu i\Delta_{\text{cf}}(x; x')\}$
2	2	a	$-\frac{1}{4}\kappa^2 \not{\partial}'\{\partial_\mu i[S](x; x')\gamma^\mu i\Delta_{\text{cf}}(x; x')\}$
2	2	b	$-\frac{1}{4}\kappa^2\partial'_\mu\{\partial^\mu\gamma^\beta i[S](x; x')\gamma_\beta i\Delta_{\text{cf}}(x; x')\}$
2	2	c	$\frac{1}{2(D-2)}\kappa^2\partial'_\mu\{\partial i[S](x; x')\gamma^\mu i\Delta_{\text{cf}}(x; x')\}$
2	3	a	$\frac{1}{8}\kappa^2\gamma^\beta\partial^\mu i[S](x; x')\gamma_\beta\partial'_\mu i\Delta_{\text{cf}}(x; x')$
2	3	b	$\frac{1}{8}\kappa^2 \not{\partial}' i\Delta_{\text{cf}}(x; x')\partial_\mu i[S](x; x')\gamma^\mu$
2	3	c	$\frac{1}{4(D-2)}\kappa^2 \not{\partial} i[S](x; x') \not{\partial}' i\Delta_{\text{cf}}(x; x')$
2	4		$\frac{1}{(D-2)}\kappa^2 iam \not{\partial} i[S](x; x') i\Delta_{\text{cf}}(x; x')$
3	1		$\frac{(D-1)}{2(D-2)}\kappa^2\partial'_\mu\{\partial i\Delta_{\text{cf}}(x; x') i[S](x; x')\gamma^\mu\}$
3	2	a	$-\frac{1}{8}\kappa^2 \not{\partial}'\{i[S](x; x') \not{\partial} i\Delta_{\text{cf}}(x; x')\}$
3	2	b	$-\frac{1}{4(D-2)}\kappa^2\partial'_\mu\{\partial i\Delta_{\text{cf}}(x; x') i[S](x; x')\gamma^\mu\}$
3	2	c	$-\frac{1}{8}\kappa^2\partial'_\mu\{\gamma^\beta i[S](x; x')\gamma_\beta\partial^\mu i\Delta_{\text{cf}}(x; x')\}$
3	3	a	$\frac{1}{16}\kappa^2\gamma^\beta i[S](x; x')\gamma_\beta\partial^\mu\partial'_\mu i\Delta_{\text{cf}}(x; x')$
3	3	b	$\frac{1}{16}\kappa^2\gamma^\mu i[S](x; x')\partial'_\mu\partial i\Delta_{\text{cf}}(x; x')$
3	3	c	$-\frac{(2D-3)}{8(D-2)}\kappa^2\gamma^\mu i[S](x; x')\partial_\mu\partial' i\Delta_{\text{cf}}(x; x')$
3	4		$-\frac{(D-1)}{2(D-2)}\kappa^2 iam \not{\partial} i\Delta_{\text{cf}}(x; x') i[S](x; x')$
4	1		$\frac{D}{(D-2)}\kappa^2 iam \partial'_\mu\{i[S](x; x')\gamma^\mu i\Delta_{\text{cf}}(x; x')\}$
4	2		$-\frac{1}{(D-2)}\kappa^2 iam \partial'_\mu\{i[S](x; x')\gamma^\mu i\Delta_{\text{cf}}(x; x')\}$
4	3		$-\frac{(D-1)}{2(D-2)}\kappa^2 iam i[S](x; x') \not{\partial} i\Delta_{\text{cf}}(x; x')$
4	4		$\frac{D}{(D-2)}\kappa^2 ia^2 m^2 i[S](x; x') \Delta_{\text{cf}}(x; x')$

Table 6: Contractions from the $i\Delta_{\text{cf}}$ part of the Graviton Propagator.

		Coefficient	Coefficient	Coefficient
I	J	$\partial^2 \frac{1}{\Delta x^{2D-4}}$	$Ha\gamma^0 \not{\partial} \frac{1}{\Delta x^{2D-4}}$	$Ha'\not{\partial}\gamma^0 \frac{1}{\Delta x^{2D-4}}$
1	4	0	0	0
2	4	0	0	0
3	4	$-\frac{1}{4} \frac{(D-1)}{(D-2)^2}$	$-\frac{1}{8} \frac{(D-1)}{(D-2)}$	0
4	1	$-\frac{1}{2} \frac{D}{(D-2)^2}$	0	$\frac{1}{4} \frac{D}{(D-2)}$
4	2	$\frac{1}{2} \frac{1}{(D-2)^2}$	0	$-\frac{1}{4} \frac{1}{(D-2)}$
4	3	$-\frac{1}{4} \frac{(D-1)}{(D-2)^2}$	$\frac{1}{8} \frac{(D-1)}{(D-2)}$	0

Table 7: $i\Delta_{\text{cf}}(x; x') \times i[S]_{\text{cf}}(x; x')$. All contributions are multiplied by $\frac{\kappa^2}{8\pi^D} \Gamma(\frac{D}{2})\Gamma(\frac{D}{2}-1)ma(aa')^{1-\frac{D}{2}}$.

The individual result is quoted in Table 7 and collect all terms of this class,

$$\begin{aligned} & \frac{\kappa^2 ma(aa')^{1-\frac{D}{2}}}{8\pi^D} \Gamma(\frac{D}{2})\Gamma(\frac{D}{2}-1) \\ & \times \frac{(D-1)}{(D-2)} \left\{ \frac{-1}{(D-2)} \partial^2 - \frac{H}{4} (a-a') \partial_0 - \frac{Ha'}{4} \gamma^0 \not{\partial} \right\} \frac{1}{\Delta x^{2D-4}}. \end{aligned} \quad (109)$$

The expression (109) is still logarithmically divergent in $D=4$ after pulling out various derivatives. To further renormalize this divergence we extract derivatives with respect to the coordinate x^μ again, which can of course be taken outside the integral in (1) to give a less singular integrand,

$$\frac{1}{\Delta x^{2D-4}} = \frac{1}{2(D-3)(D-4)} \partial^2 \frac{1}{\Delta x^{2D-6}}. \quad (110)$$

Expression (110) is integrable in four dimensions and we could take $D=4$ except for the explicit factor of $1/(D-4)$. Of course that is how ultraviolet divergences manifest in dimensional regularization. We can segregate the divergence on a local term by employing a simple representation for a delta function,

$$\frac{\partial^2}{D-4} \left(\frac{1}{\Delta x^{2D-6}} \right) = \frac{\partial^2}{D-4} \left\{ \frac{1}{\Delta x^{2D-6}} - \frac{\mu^{D-4}}{\Delta x^{D-2}} \right\} + \frac{i4\pi^{\frac{D}{2}} \mu^{D-4}}{\Gamma(\frac{D}{2}-1)} \frac{\delta^D(x-x')}{D-4},$$

$$= -\frac{\partial^2}{2} \left\{ \mu^{2D-8} \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} + O(D-4) \right\} + \frac{i4\pi^{\frac{D}{2}} \mu^{D-4}}{\Gamma(\frac{D}{2}-1)} \frac{\delta^D(x-x')}{D-4}. \quad (111)$$

After substituting (110) and (111) into (109) one can get,

$$\begin{aligned} & \frac{3\kappa^2}{64\pi^4} \left\{ \frac{1}{2} \frac{m}{a'} \partial^2 + \frac{mH}{4} \left(\frac{a}{a'} - 1 \right) \partial_0 + \frac{mH}{4} \gamma^0 \bar{\partial} \right\} \partial^2 \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] + \frac{\kappa^2}{8\pi^{\frac{D}{2}}} \Gamma\left(\frac{D}{2}\right) \frac{(aa')^{1-\frac{D}{2}}}{(D-2)} \\ & \frac{(D-1)}{(D-3)} \frac{\mu^{D-4}}{(D-4)} \left\{ \frac{-2}{(D-2)} \frac{m}{a'} \partial^2 - \frac{mH}{2} \left(\frac{a}{a'} - 1 \right) \partial_0 - \frac{mH}{2} \gamma^0 \bar{\partial} \right\} i \delta^D(x-x'). \end{aligned} \quad (112)$$

To reach the same expressions as the counterterms we mentioned in the section 3 we make use of the following identities,

$$\begin{aligned} \frac{1}{a'} \partial^2 \delta^D(x-x') &= \left\{ \frac{1}{a} \partial^2 + 2H \partial_0 \right\} \delta^D(x-x'), \\ \frac{1}{a'} \partial_0 \delta^D(x-x') &= \left\{ \frac{1}{a} \partial_0 - H \right\} \delta^D(x-x'), \\ \ln(a') \partial_0 \delta^D(x-x') &= \left\{ \ln(a) \partial_0 + Ha \right\} \delta^D(x-x'), \\ \frac{\ln(a')}{a'} \partial_0 \delta^D(x-x') &= \left\{ \frac{\ln(a)}{a} \partial_0 + H(1 - \ln(a)) \right\} \delta^D(x-x'), \\ \frac{\ln(a')}{a'} \partial^2 \delta^D(x-x') &= \left\{ \frac{\ln(a)}{a} \partial^2 + 2H(\ln(a) - 1) \partial_0 + H^2 a \right\} \delta^D(x-x'). \end{aligned} \quad (113)$$

After applying (113) to (112) and expanding out $(aa')^{1-\frac{D}{2}}$ we get the total of this most singular class which is consistent with our counterterm convention,

$$\begin{aligned} -i \left[\Sigma^{\text{cfef}} \right] (x; x') &= \frac{i\kappa^2}{16\pi^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2}-1)(D-1)\mu^{D-4}}{(D-3)(D-4)} \left\{ \frac{-2}{(D-2)} \frac{m}{a} \partial^2 - \frac{4mH}{(D-2)} \partial_0 \right. \\ & \quad \left. - \frac{1}{2} mH \gamma^0 \bar{\partial} + \frac{1}{2} mH^2 a \right\} \delta^D(x-x') + \frac{i\kappa^2}{8\pi^2} \left\{ \ln(a) \left[\frac{3}{2} \frac{m}{a} \partial^2 + 3mH \partial_0 \right. \right. \\ & \quad \left. \left. + \frac{3}{4} mH \gamma^0 \bar{\partial} - \frac{3}{4} mH^2 a \right] - \frac{3}{2} mH \partial_0 + \frac{3}{4} mH^2 a \right\} \delta^4(x-x') \\ & \quad + \frac{\kappa^2}{64\pi^2} \left\{ \frac{3}{2} \frac{m}{a'} \partial^2 + \frac{3}{4} mH \left(\frac{a}{a'} - 1 \right) \partial_0 + \frac{3}{4} mH \gamma^0 \bar{\partial} \right\} \partial^2 \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right]. \end{aligned} \quad (114)$$

A less singular contribution comes from the flat spacetime mass term of the fermion propagator. Note that the contraction (3-3) involves two derivatives acting upon the conformal graviton propagator, which would produce

$(\text{I-J})_{\text{sub}}$	$\partial^2 \frac{1}{\Delta x^{2D-4}}$	$Ha \partial_0 \frac{1}{\Delta x^{2D-4}}$	$\gamma^0 \bar{\partial} \frac{Ha}{\Delta x^{2D-4}}$	$\partial_0 \frac{Ha'}{\Delta x^{2D-4}}$	$\gamma^0 \bar{\partial} \frac{Ha'}{\Delta x^{2D-4}}$	$\frac{H^2 aa'}{\Delta x^{2D-4}}$
(1-1)	$\frac{D}{(D-2)^2}$	$\frac{-2D}{(D-2)^2}$	$\frac{-2D}{(D-2)^2}$	$\frac{-D}{2(D-2)}$	$\frac{D}{2(D-2)}$	$\frac{-D}{(D-2)}$
(1-2)	$\frac{-1}{(D-2)^2}$	$\frac{2}{(D-2)^2}$	$\frac{2}{(D-2)^2}$	$\frac{1}{2(D-2)}$	$\frac{-1}{2(D-2)}$	$\frac{1}{(D-2)}$
(1-3)	$\frac{-(D-1)}{2(D-2)^2}$	$\frac{(D-1)}{2(D-2)^2}$	$\frac{(D-1)}{2(D-2)^2}$	$\frac{(D-1)}{4(D-2)}$	$\frac{-(D-1)}{4(D-2)}$	$\frac{(D-1)}{2(D-2)}$
(2-1)	$\frac{-1}{(D-2)^2}$	$\frac{2}{(D-2)^2}$	$\frac{2}{(D-2)^2}$	$\frac{1}{2(D-2)}$	$\frac{-1}{2(D-2)}$	$\frac{1}{(D-2)}$
(2-2) _a	$\frac{-1}{4(D-2)}$	$\frac{1}{2(D-2)}$	$\frac{-1}{2(D-2)}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$
(2-2) _b	$\frac{-D}{4(D-2)}$	$\frac{D}{2(D-2)}$	0	$\frac{D}{8}$	0	$\frac{D}{4}$
(2-2) _c	$\frac{1}{2(D-2)^2}$	$\frac{-1}{(D-2)^2}$	$\frac{-1}{(D-2)^2}$	$\frac{-1}{4(D-2)}$	$\frac{1}{4(D-2)}$	$\frac{-1}{2(D-2)}$
(2-3) _a	$\frac{D}{8(D-2)}$	$\frac{-D}{8(D-2)}$	0	$\frac{-D}{16}$	0	$\frac{-D}{8}$
(2-3) _b	$\frac{1}{4(D-2)^2}$	$\frac{-1}{4(D-2)^2}$	$\frac{-1}{4(D-2)^2}$	$\frac{-1}{8(D-2)}$	$\frac{1}{8(D-2)}$	$\frac{-1}{4(D-2)}$
(2-3) _c	$\frac{1}{8(D-2)}$	$\frac{-1}{8(D-2)}$	$\frac{1}{8(D-2)}$	$\frac{-1}{16}$	$\frac{-1}{16}$	$\frac{-1}{8}$
(3-1)+(3-2) _b	$\frac{(2D-3)}{4(D-2)^2}$	$\frac{(2D-3)}{4(D-2)}$	$\frac{(2D-3)}{4(D-2)}$	$\frac{-(2D-3)}{8(D-2)}$	$\frac{(2D-3)}{8(D-2)}$	$\frac{(2D-3)}{8}$
(3-2) _a	$\frac{-1}{8(D-2)}$	$\frac{-1}{8}$	$\frac{1}{8}$	$\frac{1}{16(D-2)}$	$\frac{1}{16(D-2)}$	$\frac{-1}{16}$
(3-2) _c	$\frac{-D}{8(D-2)}$	$\frac{-D}{8}$	0	$\frac{D}{16}$	0	$\frac{-D(D-2)}{16}$
(3-3) _a	0	$\frac{D}{32}$	0	$\frac{-D}{32}$	0	$\frac{D(D-2)}{32}$
(3-2) _b	0	$\frac{1}{32}$	$\frac{-1}{32}$	$\frac{-1}{32}$	$\frac{-1}{32}$	$\frac{(D-2)}{32}$
(3-3) _c	0	$\frac{-(2D-3)}{16(D-2)}$	$\frac{-(2D-3)}{16(D-2)}$	$\frac{(2D-3)}{16(D-2)}$	$\frac{-(2D-3)}{16(D-2)}$	$\frac{-(2D-3)}{16}$

Table 8: $i\Delta_{\text{cf}} \times i[S]_{\text{fm}}$. Note that all contributions are multiplied by the factor $\frac{\kappa^2}{16\pi^D} \Gamma(\frac{D}{2}) \Gamma(\frac{D}{2} - 1) m a (aa')^{1-\frac{D}{2}}$.

a delta function,

$$\partial^\mu \partial'_\mu i\Delta_{\text{cf}} = \frac{-\Gamma(\frac{D}{2})}{4\pi^{\frac{D}{2}}} \frac{(D-2)}{(aa')^{\frac{D}{2}-1}} \left[\frac{H^2 aa' \Delta \eta^2}{\Delta x^D} + \frac{H^2 aa'}{2\Delta x^{D-2}} + \frac{i2\pi^{\frac{D}{2}}}{\Gamma(\frac{D}{2})} \delta^D(x-x') \right]. \quad (115)$$

This delta function would give zero when it is multiplied by D powers of the coordinate separation, which occurs in this case. Next, in order to facilitate our computation, we make use of the following identity except for (108),

$$\frac{H^2 aa' \Delta \eta^2}{\Delta x^{2D-2}} = \frac{Ha - Ha'}{2(D-2)} \partial_0 \frac{1}{\Delta x^{2D-4}}, \quad (116)$$

and breaks up $Ha(a') \not{\partial} \gamma^0$ into $Ha(a') \partial_0$ and $Ha(a') \gamma^0 \bar{\not{\partial}}$. The intermediate results are summarized in Table 8. When all terms in Table 8 are summed, we employ (110) for making the expression integrable in $D = 4$ and also segregate the ultraviolet divergences into the local terms using (111). The total result we get from this contribution is,

$$\begin{aligned} -i[\Sigma^{\text{effm}}](x; x') = & \frac{i\kappa^2 m H}{16\pi^{\frac{D}{2}}} \frac{2\Gamma(\frac{D}{2})\mu^{D-4}}{(D-3)(D-4)} \left\{ (b_2 + b_3) \partial_0 + (b_{2a} + b_{3a}) \gamma^0 \bar{\not{\partial}} \right. \\ & \left. + (b_4 - b_2) Ha \right\} \delta^D(x-x') + \frac{i\kappa^2 m H}{64\pi^2} \left\{ \ln(a) \left[\frac{1}{4} \gamma^0 \not{\partial} - \frac{1}{2} Ha \right] + \frac{1}{8} Ha \right\} \delta^4(x-x') \\ & + \frac{\kappa^2 m H}{64\pi^4} \left\{ \left[\frac{1}{8} \frac{a}{a'} \partial_0 - \frac{3}{32} \partial_0 + \frac{9}{16} \frac{a}{a'} \gamma^0 \bar{\not{\partial}} - \frac{17}{32} \gamma^0 \bar{\not{\partial}} + \frac{1}{4} Ha \right] \partial^2 \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] \right. \\ & \left. + \left[-\frac{3}{8} \frac{1}{Ha'} \partial^4 + \frac{Ha}{4} \partial_0^2 \right] \frac{1}{\Delta x^2} \right\}. \quad (117) \end{aligned}$$

b_2, b_{2a}, b_3, b_{3a} and b_4 are D dimension-dependent coefficients,

$$\begin{aligned} b_2 &= -\frac{(D-1)(D-5)}{8(D-2)^2} (D-4) - \frac{(3D-4)}{32(D-2)}; \\ b_{2a} &= \frac{(D-4)}{8(D-2)^2} + \frac{(-3D+5)}{2(D-2)^2} + \frac{(6D-17)}{16(D-2)} + \frac{3}{32}; \\ b_3 &= \frac{(4D-7)}{32(D-2)} (D-4) + \frac{(2D-5)}{16(D-2)}; \\ b_{3a} &= \frac{(D-4)}{32(D-2)} + \frac{(6D-7)}{16(D-2)}; \\ b_4 &= \left[\frac{-D+6}{32} + \frac{1}{8(D-2)} \right] (D-4) - \frac{1}{16}. \quad (118) \end{aligned}$$

$(\text{I-J})_{\text{sub}}$	$\partial_0 \frac{1}{\Delta x^{2D-4}}$	$\gamma^0 \bar{\partial} \frac{1}{\Delta x^{2D-4}}$	$\frac{Ha}{\Delta x^{2D-4}}$	$\frac{Ha'}{\Delta x^{2D-4}}$	$\frac{H^2 aa' \gamma^0 \gamma^k \Delta x_k}{\Delta x^{2D-4}}$
(1-1)	$\frac{4D}{(D-2)^2}$	$\frac{4D}{(D-2)^2}$	0	0	0
(1-2)	$\frac{-4}{(D-2)^2}$	$\frac{-4}{(D-2)^2}$	0	0	0
(1-3)	$\frac{-(D-1)}{(D-2)^2}$	$\frac{-(D-1)}{(D-2)^2}$	$\frac{-(D-1)(D-3)}{(D-2)^2}$	0	0
(2-1)	$\frac{-4}{(D-2)^2}$	$\frac{-4}{(D-2)^2}$	0	0	0
(2-2) _a	$\frac{(D-3)}{2(D-2)}$	0	$\frac{(2D-9)}{2(D-2)}$	0	0
(2-2) _b	$\frac{(D-5)}{2}$	0	$\frac{D(D-4)}{2(D-2)}$	0	0
(2-2) _c	$\frac{2}{(D-2)^2}$	$\frac{2}{(D-2)^2}$	0	0	0
(2-3) _a	$\frac{-(D-3)}{4}$	0	$\frac{-1}{4}$	$\frac{-(D-2)}{4}$	0
(2-3) _b	$\frac{1}{2(D-2)^2}$	$\frac{1}{2(D-2)^2}$	$\frac{(D-3)}{2(D-2)^2}$	0	0
(2-3) _c	$\frac{-1}{4}$	0	$\frac{-1}{4}$	$\frac{-3}{8}$	$\frac{-1}{4}$
(3-1)+(3-2) _b	$\frac{-(2D-3)}{2(D-2)}$	$\frac{-(2D-3)}{2(D-2)}$	$\frac{-(D-5)(2D-3)}{4(D-2)}$	$\frac{-(2D-3)}{4(D-2)}$	$\frac{(2D-3)}{4(D-2)}$
(3-2) _a	$\frac{-1}{4(D-2)}$	$\frac{(D-3)}{4(D-2)}$	$\frac{-(D-3)}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
(3-2) _c	$\frac{D-3}{4}$	0	$\frac{-D}{8}$	$\frac{D}{8}$	0
(3-3) _a	0	0	$\frac{-(D-2)(D-3)}{16}$	$\frac{(D-2)(D-3)}{16}$	0
(3-3) _b	0	$\frac{-1}{8}$	$\frac{(D-2)}{32}$	$\frac{(3D-8)}{32}$	$\frac{D-2}{32}$
(3-3) _c	0	$\frac{(2D-3)}{4(D-2)}$	$\frac{(2D-3)(3D-7)}{16(D-2)}$	$\frac{(2D-3)}{16}$	$\frac{-(2D-3)}{16}$

Table 9: $i\Delta_{\text{cf}} \times i[S]_{n=0}$. Note that all contributions are multiplied by the factor $\frac{\kappa^2 m H}{32\pi^D} \Gamma(\frac{D}{2}) \Gamma(\frac{D}{2} - 1) (aa')^{2-\frac{D}{2}}$.

At the next step we are going to consider contributions from the infinite series expansion of the fermion propagator. Because the series carries at least one power of mass we only need to consider diagrams which do not originate from the mass term in the Lagrangian. Because the infinite series is vastly more complicated than other parts of the fermion propagator it would be desirable to carry out the computation in $D = 4$ dimensions. Whether or not it is legitimate for us to do this entirely depends on whether this kind of contraction is integrable in four dimensions. The dimensionality of the series of the fermion propagator is $\frac{1}{\Delta x^{D-3-2n}}$ and the one from the conformal part of the graviton propagator is $\frac{1}{\Delta x^{D-2}}$. Also remember that all the terms in Table 5 which derive from two order m^0 vertices carry two derivatives. This means that the total dimensionality in this class is $\frac{1}{\Delta x^{2D-3-2n}}$. Therefore we shall separate the $n = 0$ part, which working on an arbitrary D dimension is necessary, from the rest of the infinite series expansion, which is integrable in four dimensions. Because $n = 0$ part is not integrable in $D = 4$ it worths mentioning its simplification from (34) by performing $\frac{m}{H}$ expansion for gamma functions rather than expanding it out around $D = 4$,

$$\Gamma\left(\frac{D}{2}-1 \pm i\frac{m}{H}\right) = \Gamma\left(\frac{D}{2}-1\right) \left[1 \pm i\frac{m}{H}\psi\left(\frac{D}{2}-1\right)\right] + \mathcal{O}(m^2), \quad (119)$$

$$\Gamma\left(1 \pm i\frac{m}{H}\right) = \left[1 \pm i\frac{m}{H}\psi(1)\right] + \mathcal{O}(m^2). \quad (120)$$

Here ψ 's stand for digamma functions and they cancel out completely at order m when one substitutes the above equations back to the $n = 0$ part of series,

$$\begin{aligned} i[S](x; x')_{n=0} &= \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2}-1)}{(2-\frac{D}{2})} (i\frac{m}{H})(aa')^{\frac{D}{2}-1} \left[i \not{\partial} \gamma^0 + i\left(\frac{D}{2}-1\right)Ha \right] \\ &\times \left[\Gamma\left(3-\frac{D}{2}\right)\Gamma\left(\frac{D}{2}-1\right) - \left(\frac{y}{4}\right)^{2-\frac{D}{2}} \right] = \Gamma\left(\frac{D}{2}-1\right) \left\{ \frac{mHa a'}{16\pi^{\frac{D}{2}}} \left[\frac{2\gamma^\nu \gamma^0 \Delta x_\nu}{\Delta x^{D-2}} \right. \right. \\ &\left. \left. + \frac{1}{(2-\frac{D}{2})} \frac{Ha}{\Delta x^{D-4}} \right] - \frac{mH^{D-3}(aa')^{\frac{D}{2}-1}}{(4\pi)^{\frac{D}{2}}} \Gamma\left(\frac{D}{2}\right)\Gamma\left(2-\frac{D}{2}\right)Ha \right\}. \end{aligned} \quad (121)$$

Note that the final two terms which have the same $D = 4$ limit in (121) tend to cancel out with each other. The first derivative of (121) has the same pattern,

$$\partial_\mu[S]_{n=0} = \Gamma\left(\frac{D}{2}-1\right) \left\{ \frac{mHa a'}{8\pi^{\frac{D}{2}}} \left[\frac{\gamma_\mu \gamma^0}{\Delta x^{D-2}} - (D-2) \frac{\gamma^\nu \gamma^0 \Delta x_\nu \Delta x_\mu}{\Delta x^D} + \frac{Ha \delta_\mu^0 \gamma^\nu \gamma^0 \Delta x_\nu}{\Delta x^{D-2}} \right] \right.$$

$(\text{I-J})_{\text{sub}}$	$Ha\partial_0^2 \frac{1}{\Delta x^{2D-6}}$	$\partial_0^2 \frac{Ha'}{\Delta x^{2D-6}}$	$Ha\gamma^0\partial_0\bar{\partial} \frac{1}{\Delta x^{2D-6}}$	$\gamma^0\partial_0\bar{\partial} \frac{Ha'}{\Delta x^{2D-6}}$	$\frac{H^2 a^2 Ha'}{\Delta x^{2D-6}}$
(1-1)	$\frac{2D}{(D-2)^2(D-3)}$	0	$\frac{2D}{(D-2)^2(D-3)}$	0	$\frac{2D}{(D-2)^2}$
(1-2)	$\frac{-2}{(D-2)^2(D-3)}$	0	$\frac{-2}{(D-2)^2(D-3)}$	0	$\frac{-2}{(D-2)^2}$
(1-3)	$\frac{-(D-1)}{2(D-2)^2(D-3)}$	0	$\frac{-(D-1)}{2(D-2)^2(D-3)}$	0	0
(2-1)	$\frac{-2}{(D-2)^2(D-3)}$	0	$\frac{-2}{(D-2)^2(D-3)}$	0	$\frac{-2}{(D-2)^2}$
(2-2) _a	0	0	0	0	$\frac{-1}{2(D-2)}$
(2-2) _b	$\frac{-1}{2(D-3)}$	0	0	0	$\frac{-D}{2(D-2)}$
(2-2) _c	$\frac{1}{(D-2)^2(D-3)}$	0	$\frac{1}{(D-2)^2(D-3)}$	0	$\frac{1}{D-2}$
(2-3) _a	$\frac{1}{8(D-3)}$	0	0	0	0
(2-3) _b	$\frac{1}{4(D-2)^2(D-3)}$	0	0	$\frac{1}{4(D-2)^2(D-3)}$	0
(2-3) _c	0	$\frac{1}{16(D-3)}$	0	0	0
(3-1)+(3-2) _b	$\frac{-(2D-3)}{4(D-2)(D-3)}$	0	$\frac{-(2D-3)}{4(D-2)(D-3)}$	0	$\frac{-(2D-3)}{8(D-2)}$
(3-2) _a	0	0	0	0	$\frac{1}{16}$
(3-2) _c	$\frac{D-2}{8(D-3)}$	0	0	0	$\frac{D}{16}$
(3-3) _a	$\frac{-(D-2)}{32(D-3)}$	$\frac{(D-2)}{32(D-3)}$	0	0	0
(3-3) _b	0	$\frac{1}{32(D-3)}$	0	$\frac{1}{32(D-3)}$	0
(3-3) _c	$\frac{(2D-3)}{16(D-2)(D-3)}$	0	$\frac{(2D-3)}{16(D-2)(D-3)}$	0	0

Table 10: $i\Delta_{\text{cf}} \times i[S]_{n=0}$. Note that all contributions are multiplied by the factor $\frac{\kappa^2 m H}{32\pi^D} \Gamma(\frac{D}{2}) \Gamma(\frac{D}{2} - 1) (aa')^{2-\frac{D}{2}}$.

$$\left. + \frac{Ha\Delta x_\mu}{\Delta x^{D-2}} - \frac{2H^2a^2\delta_\mu^0}{(D-4)\Delta x^{D-4}} \right] - \frac{m(H^2aa')^{\frac{D}{2}-1}}{H(4\pi)^{\frac{D}{2}}} \Gamma\left(\frac{D}{2}+1\right)\Gamma\left(2-\frac{D}{2}\right)H^2a^2\delta_\mu^0 \Big\}. \quad (122)$$

The final two terms of equations (121) and (122) would give a non-zero contribution when they are multiplied by the divergent term⁹.

The results derived from this class are lengthy and we would tabulate them separately based on their distinctive characteristics. Some contractions would produce at least one $(D-4)$ factor. One source of $(D-4)$ is from total derivatives acting upon $(aa')^{2-\frac{D}{2}}$. This factor can arise when one power of (aa') comes from the fermion propagator and the rest of it, $(aa')^{1-\frac{D}{2}}$, originates from the conformal part of the graviton propagator, i.e. (1-1), (1-2), (2-1), (2-2), (3-1), (3-2). Another source of $(D-4)$ comes from the following peculiar gamma function contraction,

$$\gamma^\beta \gamma^\nu \gamma^0 \gamma_\beta = (D-4)\gamma^0 \gamma^\nu + 2(D-2)\eta^{0\nu}, \quad (123)$$

which occurs in the contractions $(2-2)_b$, $(2-3)_a$, $(3-2)_c$ and $(3-3)_a$. We summarized the terms without $(D-4)$ ¹⁰ in Table 9 and Table 10 whereas the terms with the $(D-4)$ factor are presented in Table 11.

After segregating the divergences into the local terms, the total result from Table 9 and Table 10 is,

$$\begin{aligned} -i[\Sigma^{\text{cfm0-1}}](x;x') &= \frac{i\kappa^2 H^2}{32\pi^{\frac{D}{2}}} \frac{2\Gamma(\frac{D}{2})}{(D-3)} \frac{\mu^{D-4}}{(D-4)} \left\{ d_1 \frac{m}{H} \gamma^0 \bar{\partial} + (d_2 + d_3 + d_4) ma \right\} \\ &\times \delta^D(x-x') + \frac{i\kappa^2 H^2}{32\pi^2} \ln(a) \left[-2 \frac{m}{H} \gamma^0 \bar{\partial} + \frac{9}{4} ma \right] \delta^4(x-x') + \frac{\kappa^2 H^2}{64\pi^4} \left\{ \left[-\frac{1}{2} \frac{m}{H} \gamma^0 \bar{\partial} \right. \right. \\ &+ \left. \frac{9m(a+a')}{32} \right] \partial^2 \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] + \left[\frac{-5}{16} \frac{m}{H} \partial_0 + \frac{3}{4} ma + \frac{13}{32} ma' \right] \partial^2 \frac{1}{\Delta x^2} \\ &+ \left. \frac{9m(a+a')}{16} \partial_0^2 \frac{1}{\Delta x^2} + \frac{3m(5a+a')}{16} \gamma^0 \partial_0 \bar{\partial} \frac{1}{\Delta x^2} - \frac{1}{4} m H a a' \gamma^0 \bar{\partial} \frac{1}{\Delta x^2} \right\}. \quad (124) \end{aligned}$$

Here d_1 , d_2 , d_3 and d_4 are dimension-dependent coefficients,

$$d_1 = \frac{-(D-8)(3D-4)}{8(D-2)^2};$$

⁹One can consult the various gamma function contractions with (122) in Appendix A.

¹⁰The contractions (1-3), (2-3)_b, (2-3)_c, (3-3)_b and (3-3)_c produce no $(D-4)$ factor.

$(\text{I-J})_{\text{sub}}$	$\partial_0 \frac{1}{\Delta x^{2D-4}}$	$\gamma^0 \bar{\partial} \frac{1}{\Delta x^{2D-4}}$	$\frac{Ha}{\Delta x^{2D-4}}$	$\frac{Ha'}{\Delta x^{2D-4}}$	$\frac{H^2 aa' \gamma^0 \gamma^k \Delta x_k}{\Delta x^{2D-4}}$
(1-1)	0	0	$\frac{4D}{(D-2)^2}$	0	0
(1-2)	0	0	$\frac{-4}{(D-2)^2}$	0	0
(1-3)	0	0	0	0	0
(2-1)	0	0	$\frac{-4}{(D-2)^2}$	0	0
(2-2) _a	0	0	$\frac{-1}{(D-2)}$	$\frac{3}{2(D-2)}$	$\frac{1}{(D-2)}$
(2-2) _b	$\frac{-(D-5)}{2(D-2)}$	$\frac{-(D-5)}{2(D-2)}$	$\frac{-(3D-4)}{2(D-2)}$	$\frac{3}{2}$	0
(2-2) _c	0	0	$\frac{2}{(D-2)^2}$	0	0
(2-3) _a	$\frac{(D-3)}{4(D-2)}$	$\frac{(D-3)}{4(D-2)}$	$\frac{1}{4(D-2)}$	$\frac{3}{8}$	$\frac{1}{4}$
(2-3) _b	0	0	0	0	0
(2-3) _c	0	0	0	0	0
(3-1)+(3-2) _b	0	0	$\frac{-(2D-3)}{4(D-2)}$	$\frac{-(2D-3)}{4(D-2)}$	$\frac{(2D-3)}{4(D-2)}$
(3-2) _a	0	0	$\frac{-1}{8}$	$\frac{-(D-4)}{8(D-2)}$	$\frac{1}{8}$
(3-2) _c	$\frac{-(D-3)}{4(D-2)}$	$\frac{-(D-3)}{4(D-2)}$	$\frac{-(D-2)}{4}$	$\frac{(D-3)}{4}$	0
(3-3) _a	0	0	$\frac{(D-3)}{16}$	$\frac{-(D-3)}{16}$	$\frac{-(D-2)}{16}$
(3-3) _b	0	0	0	0	0
(3-3) _c	0	0	0	0	0

Table 11: $i\Delta_{\text{cf}} \times i[S]_{n=0}$ with a $(D-4)$ pre-factor. All contributions are multiplied by $\frac{\kappa^2 m H}{32\pi^D} \Gamma(\frac{D}{2}) \Gamma(\frac{D}{2} - 1) (aa')^{2-\frac{D}{2}} \frac{(D-4)}{2}$.

$$d_2 = -\frac{1}{4} - \frac{1}{(D-2)} + (D-4) \left[\frac{-D}{16} + \frac{9}{32} - \frac{3}{8(D-2)} \right];$$

$$d_3 = \frac{(D-6)}{8} \left[\frac{1}{2} - \frac{(2D-3)}{(D-2)^2} \right] \quad ; \quad d_4 = -\frac{3}{16}(D-1). \quad (125)$$

Because the contributions from Table 11 carries a factor of $(D-4)$, they would survive only if they combine with a $\frac{1}{(D-4)}$ after making them integrable in $D = 4$ dimensions. The result from this part is finite so we give an expression for the $D = 4$,

$$-i[\Sigma^{\text{cf}0-2}](x; x') = \frac{\kappa^2 H^2}{64\pi^4} \left\{ \frac{1}{8} \frac{m}{H} \gamma^0 \not{\partial} - \frac{17}{32} m a + \frac{35}{32} m a' \right\} \partial^2 \frac{1}{\Delta x^2}. \quad (126)$$

The contributions which originate from the final two terms of (121) and (122) tend to cancel. They could be taken special care first before they are tabulated because most of contributions from this class give finite results in $D = 4$ dimensions except for a few divergent terms from the double derivatives on $\Delta_{\text{cf}}(x; x')$. The strategy to deal with the finite part of the contributions is to perform the $(D-4)$ expansion and make use of (111) and the following key identities,

$$\partial_\mu \frac{1}{\Delta x^{D-2}} = \mu^{D-4} \left[1 + \frac{1}{2}(D-4)(1 - \ln \Delta x^2) \right] \partial_\mu \frac{1}{\Delta x^2}, \quad (127)$$

$$\partial_\mu \frac{1}{\Delta x^{2D-6}} = \mu^{2D-8} \left[1 + \frac{1}{2}(D-4)(2 - 2 \ln \Delta x^2) \right] \partial_\mu \frac{1}{\Delta x^2}, \quad (128)$$

$$\partial^2 \frac{1}{\Delta x^{D-2}} = \frac{i 4 \pi^{\frac{D}{2}}}{\Gamma(\frac{D}{2}-1)} \delta^D(x-x'). \quad (129)$$

Here we present one example from $(3-2)_c$,

$$\Gamma\left(\frac{D}{2}\right) \Gamma\left(\frac{D}{2}-1\right) \left(\frac{m}{H}\right) \frac{\kappa^2}{8} \left\{ \frac{H^2 (aa')^{2-\frac{D}{2}}}{32\pi^{\frac{D}{2}}(D-4)} \left[\frac{DHa}{2(D-3)} \partial^2 + \frac{D}{2} H^2 a^2 \partial_0 \right] \frac{1}{\Delta x^{2D-6}} \right.$$

$$\left. - \frac{H^{D-2}}{2^{D+1}\pi^D} \Gamma\left(\frac{D}{2}\right) \frac{\Gamma(3-\frac{D}{2})}{\frac{1}{2}(D-4)} \left[\frac{DHa}{(D-2)} \partial^2 + \frac{D}{2} H^2 a^2 \partial_0 \right] \frac{1}{\Delta x^{D-2}} \right\}. \quad (130)$$

There are three distinctive contributions in (130). We extract out the pre-factor $\Gamma(\frac{D}{2}) \Gamma(\frac{D}{2}-1) \frac{\kappa^2 m H}{64\pi^D}$ to avoid the repeated and lengthy expressions. The first kind comes from the two single derivative terms,

$$\frac{1}{2} \mu^{2D-8} \frac{DH^2 a^2}{2} \left\{ \left[1 + \frac{1}{2}(D-4)(-\ln aa' + 2 - 2 \ln \mu^2 \Delta x^2) \right] + \left[-1 + \frac{1}{2}(D-4) \right] \right\}$$

$(\text{I-J})_{\text{sub}}$	$H^2 a^2 \partial_0 \frac{1}{\Delta x^2}$	$\gamma^0 \bar{\partial} \frac{H^2 a'^2}{\Delta x^2}$	$\frac{H^2 a^2 H a'}{\Delta x^2}$	$H a \partial^2 \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right]$
(1-1)	$-2 - 4 \ln(\frac{y}{4})$	$-2 - 4 \ln(\frac{y}{4})$	0	0
(1-2)	$\frac{1}{2} + \ln(\frac{y}{4})$	$\frac{1}{2} + \ln(\frac{y}{4})$	0	0
(1-3)	$\frac{9}{4} + \frac{3}{2} \ln(\frac{y}{4})$	$\frac{9}{4} + \frac{3}{2} \ln(\frac{y}{4})$	$\frac{9}{4} + \frac{3}{2} \ln(\frac{y}{4})$	0
(2-1)	$\frac{1}{2} + \ln(\frac{y}{4})$	$\frac{1}{2} + \ln(\frac{y}{4})$	0	0
(2-2) _a	$\frac{1}{4} + \frac{1}{2} \ln(\frac{y}{4})$	$-\frac{1}{4} - \frac{1}{2} \ln(\frac{y}{4})$	0	0
(2-2) _b	$1 + 2 \ln(\frac{y}{4})$	0	0	0
(2-2) _c	$-\frac{1}{4} - \frac{1}{2} \ln(\frac{y}{4})$	$-\frac{1}{4} - \frac{1}{2} \ln(\frac{y}{4})$	0	0
(2-3) _a	$-\frac{3}{2} - \ln(\frac{y}{4})$	0	$-\frac{3}{2} - \ln(\frac{y}{4})$	0
(2-3) _b	$-\frac{3}{8} - \frac{1}{4} \ln(\frac{y}{4})$	$-\frac{3}{8} - \frac{1}{4} \ln(\frac{y}{4})$	$-\frac{3}{8} - \frac{1}{4} \ln(\frac{y}{4})$	0
(2-3) _c	$-\frac{3}{8} - \frac{1}{4} \ln(\frac{y}{4})$	$\frac{3}{8} + \frac{1}{4} \ln(\frac{y}{4})$	$-\frac{3}{8} - \frac{1}{4} \ln(\frac{y}{4})$	0
(3-1)+(3-2) _b	$\frac{5}{8} \ln(\frac{y}{4})$	$\frac{5}{8} \ln(\frac{y}{4})$	0	$\frac{5}{8}$
(3-2) _a	$-\frac{1}{8} \ln(\frac{y}{4})$	$-\frac{1}{8} \ln(\frac{y}{4})$	0	$-\frac{1}{8}$
(3-2) _c	$-\frac{1}{2} \ln(\frac{y}{4})$	0	0	$-\frac{1}{2}$
Sub-total	0	$\frac{3}{4} - \ln(\frac{y}{4})$	0	0

Table 12: $i\Delta_{\text{cf}} \times i[S]_{n=0}$ form the two terms which tend to cancel. All contributions are multiplied by $\frac{\kappa^2 m H}{64\pi^4}$.

$(\text{I-J})_{\text{sub}}$	$\frac{H^2 a^2 \Delta \eta}{\Delta x^4}$	$\frac{H^2 a a' \Delta \eta}{\Delta x^4}$	$\frac{H^2 a^2 \gamma^0 \gamma^k \Delta x_k}{\Delta x^4}$	$\frac{H^2 a a' \gamma^0 \gamma^k \Delta x_k}{\Delta x^4}$	$\frac{H^2 a^2 H a'}{\Delta x^2}$
$(3-3)_a$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{4}$
$(3-3)_b$	$\frac{1}{8}$	$-\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{16}$
$(3-3)_c$	$-\frac{5}{8}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{-5}{16}$
Sub-total	0	0	$\frac{3}{4}$	$\frac{3}{4}$	0

Table 13: $i\Delta_{\text{cf}} \times i[S]_{n=0}$ from the two terms which tend to cancel. All contributions are multiplied by $\frac{\kappa^2 m H}{64\pi^4} \times \left(1 + \ln\left(\frac{y}{4}\right)\right)$.

$$\times \left(-\ln \frac{H^2}{4\mu^2} - 1 - 1 + \ln \mu^2 \Delta x^2\right) \Big] \partial_0 \frac{1}{\Delta x^2} \longrightarrow \frac{D H^2 a^2}{8} \left[-\ln \frac{y}{4}\right] \partial_0 \frac{1}{\Delta x^2}, \quad (131)$$

and we presented the temporal and spatial contributions separately in Table 12. The second kind is the contributions which produce a delta function originated from the two double derivative terms,

$$\begin{aligned} & \frac{\frac{1}{4}\mu^{D-4}}{\frac{1}{2}(D-4)} \frac{D H a}{2} \left\{ \left[1 + \frac{1}{2}(D-4)(-\ln a a' - 2)\right] + \left[-1 + \frac{1}{2}(D-4)\left(-\ln \frac{H^2}{4\mu^2} - 1 + 1\right)\right] \right\} \\ & \times \frac{i 4\pi^{\frac{D}{2}} \delta^D(x-x')}{\Gamma(\frac{D}{2}-1)} \longrightarrow -D H a \left[\ln \frac{H a}{2\mu} + 1\right] \frac{i \pi^{\frac{D}{2}} \mu^{D-4}}{\Gamma(\frac{D}{2}-1)} \delta^D(x-x'), \end{aligned} \quad (132)$$

and the final results are displayed in the first column of Table 14. The third kind is the residual term from $\partial^2 \frac{1}{\Delta x^{2D-6}}$,

$$\frac{\mu^{2D-8}}{\frac{1}{2}(D-4)} \times -\frac{1}{2}(D-4) \frac{D H a}{8} \partial^2 \left[\frac{\ln \mu^2 \Delta x^2}{\Delta x^2}\right], \quad (133)$$

and we give the results in the final column of Table 12.

The final contribution in this category is from $(3-3)$ which consists of some finite terms and local divergent terms. Recall in (115) that two derivatives acting on the conformal part of the graviton propagator would produce a delta function. In general, it would be zero in dimensional regularization when it acts on dimension-dependent power of the coordinate separation. However, the last term with the divergent coefficient in (121) does not possess any dimension-dependent power of Δx^2 . As a result, when $i[S]_{n=0}$ are

$(\text{I-J})_{\text{sub}}$	$\frac{i\kappa^2 H^2}{64\pi^2} ma\delta^4(x-x')$	$\frac{i\kappa^2 H^{D-2}}{2^{D+2}\pi^{\frac{D}{2}}}\Gamma(\frac{D}{2})\frac{\Gamma(3-\frac{D}{2})}{\frac{1}{2}(D-4)}ma\delta^D(x-x')$
$(3-1)+(3-2)_b$	$5[1 + \ln(\frac{Ha}{2\mu})]$	0
$(3-2)_a$	$-[1 + \ln(\frac{Ha}{2\mu})]$	0
$(3-2)_c$	$-4[1 + \ln(\frac{Ha}{2\mu})]$	0
$(3-3)_a$	0	$\Gamma(\frac{D}{2}) \times \frac{D}{4}$
$(3-3)_b$	0	$\Gamma(\frac{D}{2}) \times (1 - \frac{D}{2})$
$(3-3)_c$	0	$\Gamma(\frac{D}{2} - 1) \times \frac{(2D-3)}{2}$

Table 14: $i\Delta_{\text{cf}} \times i[S]_{\text{n}=0}$. The delta function contribution from the two terms which tend to cancel.

multiplied by $\partial.\partial\Delta_{\text{cf}}$, no any other terms in the calculation can be used to cancel this particular divergent local term. Here we present $(3-3)_a$ as an example,

$$\begin{aligned} & \Gamma(\frac{D}{2})\Gamma(\frac{D}{2}-1)\frac{-(D-2)}{16}\frac{m}{H}\frac{\kappa^2}{2}\left\{\frac{H^2(aa')^{2-\frac{D}{2}}}{32\pi^D\frac{1}{2}(D-4)}\left[\frac{DH^3a^2a'\Delta\eta^2}{\Delta x^{2D-4}}+\frac{D}{2}\frac{H^3a^2a'}{\Delta x^{2D-6}}\right]\right. \\ & \left.-\frac{H^{D-2}\Gamma(\frac{D}{2})\Gamma(3-\frac{D}{2})}{2^{D+1}\pi^D}\frac{1}{\frac{1}{2}(D-4)}\left[\frac{DH^3a^2a'\Delta\eta^2}{\Delta x^D}+\frac{D}{2}\frac{H^3a^2a'}{\Delta x^{D-2}}+\frac{DHai2\pi^{\frac{D}{2}}}{\Gamma(\frac{D}{2})}\delta^D(x-x')\right]\right\}. \end{aligned} \quad (134)$$

We employ the same trick to deal with the finite part, extract out the prefactor $\Gamma(\frac{D}{2})\Gamma(\frac{D}{2}-1)\frac{\kappa^2 m H}{64\pi^D}\frac{-(D-2)}{16}$,

$$\begin{aligned} & \left\{\left[1+\frac{1}{2}(D-4)(-\ln aa'-2\ln \mu^2\Delta x^2)\right]+\left[-1+\frac{1}{2}(D-4)(-\ln \frac{H^2}{4\mu^2}-1+\ln \mu^2\Delta x^2)\right]\right\} \\ & \times \frac{\mu^{2D-8}}{\frac{1}{2}(D-4)}\left[\frac{DH^3a^2a'\Delta\eta^2}{\Delta x^2}+\frac{D}{2}\frac{H^3a^2a'}{\Delta x^2}\right], \end{aligned} \quad (135)$$

and tabulate the result for $D = 4$ in Table 13. The divergent term can be read off directly and we present it in the second column of Table 13. Finally we enclose this sub-class by summing up all the terms from Tables 12, 13

and 14,

$$\begin{aligned}
-i[\Sigma^{\text{cfno-3}}](x; x') &= \frac{i\kappa^2 H^{D-2}}{2^{D+1}\pi^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2})\Gamma(\frac{D}{2}-1)\Gamma(3-\frac{D}{2})(2D-3)}{2(D-4)} ma\delta^D(x-x') \\
&\quad - \frac{i\kappa^2 H^2}{32\pi^2} \frac{ma}{4} \delta^4(x-x') + \frac{\kappa^2 H^2}{64\pi^4} \left\{ \left[\frac{3}{8} - \frac{11}{8} \ln\left(\frac{y}{4}\right) \right] mHa^2 \right. \\
&\quad \left. - \frac{3}{8} \left[1 + \ln\left(\frac{y}{4}\right) \right] mHaa' \right\} \gamma^0 \bar{\not{D}} \frac{1}{\Delta x^2}. \quad (136)
\end{aligned}$$

The last computation in this sub-section involves the rest of the infinite series expansion which are all integrable and hence we can compute it in $D = 4$ dimensions directly. The fermion propagator for the $n \geq 1$ of the series in four dimensions we employed is,

$$i[S]_{n \geq 1} = \frac{mHaa'}{16\pi^2} \sum_{n=1}^{\infty} \left\{ \frac{2\gamma^\nu \gamma^0 \Delta x_\nu}{\Delta x^2} \left[1 + n \ln\left(\frac{y}{4}\right) \right] + Ha \left[1 + (n+1) \ln\left(\frac{y}{4}\right) \right] \right\} \left(\frac{y}{4}\right)^n, \quad (137)$$

and its derivative¹¹ is,

$$\begin{aligned}
\partial_\mu i[S]_{n \geq 1} &= \frac{mHaa'}{16\pi^2} \sum_{n=1}^{\infty} \left(\frac{y}{4}\right)^n \left\{ \frac{4\gamma^\nu \gamma^0 \Delta x_\nu \Delta x_\mu}{\Delta x^4} \left[(2n-1) + (n^2-n) \ln\frac{y}{4} \right] \right. \\
&\quad + \frac{2\gamma_\mu \gamma^0}{\Delta x^2} \left[1 + n \ln\frac{y}{4} \right] + \frac{2Ha(\delta_\mu^0 \gamma^\nu \gamma^0 \Delta x_\nu + \Delta x_\mu)}{\Delta x^2} \left[(2n+1) + (n^2+n) \ln\frac{y}{4} \right] \\
&\quad \left. + H^2 a^2 \delta_\mu^0 \left[(2n+3) + (n^2+3n+2) \ln\frac{y}{4} \right] \right\}. \quad (138)
\end{aligned}$$

One interesting pattern is that taking the derivative of the coefficient of the logarithm term with respect to n gives the coefficient for the term without logarithms. Before we table the result from each length expression, we present the result from the contraction (1-1),

$$\begin{aligned}
2\kappa^2 \partial'_\mu \{ \not{D} i[S](x; x') \gamma^\mu i\Delta_{\text{cf}}(x; x') \} &= \frac{\kappa^2 mH}{32\pi^4} \left\{ 8 \left[\frac{\gamma^0 \gamma^\mu \Delta x_\mu}{\Delta x^6} + \frac{Ha \gamma^0 \gamma^\mu \Delta \eta \Delta x_\mu}{\Delta x^6} \right] \right. \\
&\quad \times \left[(3n^2 - 2n - 2) + (n^3 - n^2 - 2n) \ln\left(\frac{y}{4}\right) \right] - \frac{4Ha}{\Delta x^4} \left[(3n^2 - 1) + (n^3 - n) \ln\left(\frac{y}{4}\right) \right] \\
&\quad - \frac{2H^2 a^2 \gamma^0 \gamma^\mu \Delta x_\mu}{\Delta x^4} \left[(3n^2 + 4n - 1) + (n^3 + 2n^2 - n - 2) \ln\left(\frac{y}{4}\right) \right] \\
&\quad \left. + \frac{H^2 a^2 Ha'}{\Delta x^4} \left[(3n^2 + 6n + 2) + (n^3 + n^2 + 2n) \ln\left(\frac{y}{4}\right) \right] \right\} \left(\frac{y}{4}\right)^n. \quad (139)
\end{aligned}$$

¹¹One can find various gamma functions contracted with (138) in Appendix A

$(\text{I-J})_{\text{sub}}$	$\frac{\gamma^0 \gamma^k \Delta x_k}{\Delta x^6}$	$\frac{\Delta \eta}{\Delta x^6}$	$\frac{Ha \gamma^0 \Delta \eta \gamma^k \Delta x_k}{\Delta x^6}$	$\frac{Ha' \gamma^0 \Delta \eta \gamma^k \Delta x_k}{\Delta x^6}$	$\frac{Ha \Delta \eta^2}{\Delta x^6}$	$\frac{Ha' \Delta \eta^2}{\Delta x^6}$
(1-1)	$16(3n^2-2n-2)$	$-16(3n^2-2n-2)$	$16(3n^2-2n-2)$	0	$-16(3n^2-2n-2)$	0
(1-2)	$-4(3n^2-2n-2)$	$4(3n^2-2n-2)$	$-4(3n^2-2n-2)$	0	$4(3n^2-2n-2)$	0
(1-3)	$6(2n+1)$	$-6(2n+1)$	$6(2n+1)$	0	$-6(2n+1)$	0
(2-1)	$-4(3n^2-2n-2)$	$4(3n^2-2n-2)$	$-4(3n^2-2n-2)$	0	$4(3n^2-2n-2)$	0
(2-2) _a	$-2(3n^2-2n-1)$	$2(3n^2-2n+1)$	0	$2(3n^2-2n)$	0	$-2(3n^2-2n)$
(2-2) _b	0	$4(6n^2-4n-1)$	0	0	$4(3n^2-2n-2)$	$-4(3n^2-2n)$
(2-2) _c	$2(3n^2-2n-2)$	$-2(3n^2-2n-2)$	$2(3n^2-2n-2)$	0	$-2(3n^2-2n-2)$	0
(2-3) _a	0	$2(4n-1)$	0	0	$2(2n+1)$	$-2(2n-1)$
(2-3) _b	$-(2n+1)$	$(2n+1)$	$-(2n+1)$	0	$(2n+1)$	0
(2-3) _c	$-2n$	$(2n-2)$	0	$(2n-1)$	0	$-(2n-1)$
(3-1)	$-6(2n-2)$	$6(2n-2)$	$-6(2n-2)$	0	$6(2n-2)$	0
(3-2) _a	$(2n-1)$	$-(2n+1)$	0	$-2n$	0	$2n$
(3-2) _b	$(2n-2)$	$-(2n-2)$	$(2n-2)$	0	$-(2n-2)$	0
(3-2) _c	0	$-2(4n-1)$	0	0	$-2(2n-2)$	$4n$
(3-3) _a	0	0	0	0	-1	+1
(3-3) _b	1	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$
(3-3) _c	-5	0	$-\frac{5}{2}$	0	$\frac{5}{2}$	0
Total _{sub}	$8(3n^2-2n-1)$	0	$5f'(n)$	$f'(n)$	$-3f'(n)$	$-3f'(n)$

Table 15: $i\Delta_{\text{cf}} \times i[S]_{n \geq 1} - \text{I}$. All contributions are multiplied by $\frac{\kappa^2 m H}{64\pi^4} \sum_{n \geq 1} \left(\frac{y}{4}\right)^n$. Here $f'(n) = (6n^2 - 4n - \frac{3}{2})$.

$(\text{I-J})_{\text{sub}}$	$\frac{\gamma^0 \gamma^k \Delta x_k}{\Delta x^6}$	$\frac{\Delta \eta}{\Delta x^6}$	$\frac{H a \gamma^0 \Delta \eta \gamma^k \Delta x_k}{\Delta x^6}$	$\frac{H a' \gamma^0 \Delta \eta \gamma^k \Delta x_k}{\Delta x^6}$	$\frac{H a \Delta \eta^2}{\Delta x^6}$	$\frac{H a' \Delta \eta^2}{\Delta x^6}$
(1-1)	$16(n^3 - n^2 - 2n)$	$-16(n^3 - n^2 - 2n)$	$16(n^3 - n^2 - 2n)$	0	$-16(n^3 - n^2 - 2n)$	0
(1-2)	$-4(n^3 - n^2 - 2n)$	$4(n^3 - n^2 - 2n)$	$-4(n^3 - n^2 - 2n)$	0	$4(n^3 - n^2 - 2n)$	0
(1-3)	$6(n^2 + n)$	$-6(n^2 + n)$	$6(n^2 + n)$	0	$-6(n^2 + n)$	0
(2-1)	$-4(n^3 - n^2 - 2n)$	$4(n^3 - n^2 - 2n)$	$-4(n^3 - n^2 - 2n)$	0	$4(n^3 - n^2 - 2n)$	0
(2-2) _a	$-2(n^3 - n^2 - n)$	$2(n^3 - n^2 + n)$	0	$2(n^3 - n^2)$	0	$-2(n^3 - n^2)$
(2-2) _b	0	$4(2n^3 - 2n^2 - n)$	0	0	$4(n^3 - n^2 - 2n)$	$-4(n^3 - n^2)$
(2-2) _c	$2(n^3 - n^2 - 2n)$	$-2(n^3 - n^2 - 2n)$	$2(n^3 - n^2 - 2n)$	0	$-2(n^3 - n^2 - 2n)$	0
(2-3) _a	0	$2(2n^2 - n)$	0	0	$2(n^2 + n)$	$-2(n^2 - n)$
(2-3) _b	$-(n^2 + n)$	$(n^2 + n)$	$-(n^2 + n)$	0	$(n^2 + n)$	0
(2-3) _c	$-n^2$	$(n^2 - 2n)$	0	$(n^2 - n)$	0	$-(n^2 - n)$
(3-1)	$-6(n^2 - 2n)$	$6(n^2 - 2n)$	$-6(n^2 - 2n)$	0	$6(n^2 - 2n)$	0
(3-2) _a	$(n^2 - n)$	$-(n^2 + n)$	0	$-n^2$	0	n^2
(3-2) _b	$(n^2 - 2n)$	$-(n^2 - 2n)$	$(n^2 - 2n)$	0	$-(n^2 - 2n)$	0
(3-2) _c	0	$-2(2n^2 - n)$	0	0	$-2(n^2 - 2n)$	$2n^2$
(3-3) _a	0	0	0	0	$-n$	$+n$
(3-3) _b	n	0	0	$-\frac{1}{2}n$	0	$\frac{1}{2}n$
(3-3) _c	$-5n$	0	$-\frac{5}{2}n$	0	$\frac{5}{2}n$	0
Total _{sub}	$8(n^3 - n^2 - n)$	0	$5f(n)$	$f(n)$	$-3f(n)$	$-3f(n)$

Table 16: $i\Delta_{\text{cf}} \times i[S]_{n \geq 1} - \text{I}'$. All contributions are multiplied by $\frac{\kappa^2 m H}{64\pi^4} \sum_{n \geq 1} \left(\frac{y}{4}\right)^n \ln\left(\frac{y}{4}\right)$. Here $f(n) = 2n^3 - 2n^2 - \frac{3}{2}n$.

The same pattern here happened again! We should keep it in mind that this pattern might maintain after summing up each individual contribution. We also separate temporal terms with spatial ones for our conventional choice of counterterms and summarized the results in Tables 15, 16, 17 and 18 before many summations are performed. Table 16 (Table 18) is the partner of Table 15 (Table 17) with the extra logarithm, $\ln \frac{y}{4}$. From the coefficient of each individual term at the bottom of each table one might already notice that the pattern we mentioned above still exists. The benefit for postponing the infinite summation for each individual contraction not only because it is a less complicated procedure but also because the pattern serves us one consistent check whether or not we have done the computation correctly for such a long and complicated calculation.

We can easily read off the contribution from each distinctive term. For example, the total coefficient of $\frac{\gamma^0 \gamma^k \Delta x_k}{\Delta x^6}$ from Table 15 and Table 16 is,

$$\frac{\kappa^2 m H}{64 \pi^4} \sum_{n=1}^{\infty} \left[8(3n^2 - 2n - 1) + 8(n^3 - n^2 - n) \ln\left(\frac{y}{4}\right) \right] \left(\frac{y}{4}\right)^n. \quad (140)$$

After collecting all of terms from the four tables we obtain,

$$\begin{aligned} & \frac{\kappa^2 m H}{64 \pi^4} \sum_{n=1}^{\infty} \left(\frac{y}{4}\right)^n \left\{ \frac{\gamma^0 \gamma^k \Delta x_k}{\Delta x^6} \left[8(3n^2 - 2n - 1) + 8(n^3 - n^2 - n) \ln\left(\frac{y}{4}\right) \right] \right. \\ & + \left[5 \frac{H a \gamma^0 \Delta \eta \gamma^k \Delta x_k}{\Delta x^6} + \frac{H a' \gamma^0 \Delta \eta \gamma^k \Delta x_k}{\Delta x^6} - 3 \frac{H a \Delta \eta^2}{\Delta x^6} - 3 \frac{H a' \Delta \eta^2}{\Delta x^6} \right] \left[(6n^2 - 4n - \frac{3}{2}) \right. \\ & + \left. (2n^3 - 2n^2 - \frac{3}{2}n) \ln\left(\frac{y}{4}\right) \right] - 3 \left[\frac{H a}{\Delta x^4} + \frac{H a'}{\Delta x^4} \right] \left[(6n^2 + 2n) + (2n^3 + n^2) \ln\left(\frac{y}{4}\right) \right] \\ & + \frac{H^2 a^2 \gamma^0 \gamma^k \Delta x_k}{\Delta x^4} \left[-(9n^2 + 12n + \frac{3}{4}) + (-3n^3 - 6n^2 - \frac{3}{4}n + \frac{9}{4}) \ln\left(\frac{y}{4}\right) \right] \\ & \left. + \frac{H^2 a a' \gamma^0 \gamma^k \Delta x_k}{\Delta x^4} \left[(-3n^2 - 2n + \frac{5}{4}) + (-n^3 - n^2 + \frac{5}{4}n + \frac{3}{4}) \ln\left(\frac{y}{4}\right) \right] \right\}. \quad (141) \end{aligned}$$

One might notice that the infinite series could be summed easily using the following identities,

$$\sum_{n=1}^{\infty} Y^n = \frac{Y}{1-Y} \quad ; \quad \sum_{n=1}^{\infty} n Y^n = \frac{Y}{(1-Y)^2} \quad (142)$$

$$\sum_{n=1}^{\infty} n^2 Y^n = \frac{Y(Y+1)}{(1-Y)^3} \quad ; \quad \sum_{n=1}^{\infty} n^3 Y^n = \frac{Y(Y^2 + 4Y + 1)}{(1-Y)^3}. \quad (143)$$

$(\text{I-J})_{\text{sub}}$	$\frac{Ha}{\Delta x^4}$	$\frac{Ha'}{\Delta x^4}$	$\frac{H^2 a^2 \gamma^0 \gamma^k \Delta x_k}{\Delta x^4}$	$\frac{H^2 aa' \gamma^0 \gamma^k \Delta x_k}{\Delta x^4}$	$\frac{H^2 a^2 \Delta \eta}{\Delta x^4}$	$\frac{H^2 a^2 Ha'}{\Delta x^2}$
(1-1)	$-8(3n^2-1)$	0	$-4(3n^2+4n-1)$	0	$4(3n^2+4n-1)$	$2(3n^2+6n+2)$
(1-2)	$2(3n^2-1)$	0	$(3n^2+4n-1)$	0	$-(3n^2+4n-1)$	$-\frac{1}{2}(3n^2+6n+2)$
(1-3)	$-3(2n+1)$	0	$-\frac{3}{2}(2n+3)$	0	$\frac{3}{2}(2n+3)$	$\frac{3}{4}(2n+3)$
(2-1)	$2(3n^2-1)$	0	$(3n^2+4n-1)$	0	$-(3n^2+4n-1)$	$-\frac{1}{2}(3n^2+6n+2)$
(2-2) _a	$-(3n^2+2n)$	$-2(3n^2+n)$	$-\frac{1}{2}(3n^2+4n-1)$	$-(3n^2+2n)$	$-\frac{1}{2}(3n^2+4n-1)$	$-\frac{1}{4}(3n^2+6n+2)$
(2-2) _b	$-2(2n+1)$	$-2(6n^2+2n)$	0	0	$-2(3n^2+4n-1)$	$-(3n^2+6n+2)$
(2-2) _c	$-(3n^2-1)$	0	$-\frac{1}{2}(3n^2+4n-1)$	0	$\frac{1}{2}(3n^2+4n-1)$	$\frac{1}{4}(3n^2+6n+2)$
(2-3) _a	0	$-(4n+1)$	0	0	$-(2n+3)$	$-\frac{1}{2}(2n+3)$
(2-3) _b	$\frac{1}{2}(2n+1)$	0	$\frac{1}{4}(2n+3)$	0	$-\frac{1}{4}(2n+3)$	$-\frac{1}{8}(2n+3)$
(2-3) _c	$-\frac{1}{2}(2n+1)$	$-(2n+\frac{1}{2})$	$-\frac{1}{4}(2n+3)$	$-\frac{1}{2}(2n+1)$	$-\frac{1}{4}(2n+3)$	$-\frac{1}{8}(2n+3)$
(3-1)	$6(n-\frac{3}{4})$	$-\frac{3}{2}$	$3n$	$\frac{3}{2}$	$-3n$	$-\frac{3}{4}(2n+1)$
(3-2) _a	$\frac{1}{4}(4n+1)$	$(2n+\frac{1}{4})$	$\frac{1}{2}n$	$\frac{1}{4}(4n+1)$	$\frac{1}{2}n$	$\frac{1}{8}(2n+1)$
(3-2) _b	$-(n-\frac{3}{4})$	$\frac{1}{4}$	$-\frac{1}{2}n$	$-\frac{1}{4}$	$\frac{1}{2}n$	$\frac{1}{8}(2n+1)$
(3-2) _c	2	$(4n+1)$	0	0	$2n$	$\frac{1}{2}(2n+1)$
(3-3) _a	-1	1	0	0	$\frac{1}{2}$	$\frac{1}{4}$
(3-3) _b	0	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
(3-3) _c	$\frac{5}{2}$	0	$\frac{5}{8}$	0	$-\frac{5}{8}$	$-\frac{5}{16}$
Total _{sub}	$-6(3n^2+n)$	$-6(3n^2+n)$	$-(9n^2+12n+\frac{3}{4})$	$(-3n^2-2n+\frac{5}{4})$	0	0

Table 17: $i\Delta_{\text{cf}} \times i[S]_{n \geq 1} - \text{II}$. All contributions are multiplied by $\frac{\kappa^2 m H}{64\pi^4} \sum_{n \geq 1} \left(\frac{y}{4}\right)^n$.

$(\text{I-J})_{\text{sub}}$	$\frac{Ha}{\Delta x^4}$	$\frac{Ha'}{\Delta x^4}$	$\frac{H^2 a^2 \gamma^0 \gamma^k \Delta x_k}{\Delta x^4}$	$\frac{H^2 a a' \gamma^0 \gamma^k \Delta x_k}{\Delta x^4}$	$\frac{H^2 a^2 \Delta \eta}{\Delta x^4}$	$\frac{H^2 a^2 Ha'}{\Delta x^2}$
(1-1)	$-8(n^3-n)$	0	$-4(n^3+2n^2-n-2)$	0	$4(n^3+2n^2-n-2)$	$2(n^3+3n^2+2n)$
(1-2)	$2(n^3-n)$	0	(n^3+2n^2-n-2)	0	$-(n^3+2n^2-n-2)$	$-\frac{1}{2}(n^3+3n^2+2n)$
(1-3)	$-3(n^2+n)$	0	$-\frac{3}{2}(n^2+3n+2)$	0	$\frac{3}{2}(n^2+3n+2)$	$\frac{3}{4}(n^2+3n+2)$
(2-1)	$2(n^3-n)$	0	(n^3+2n^2-n-2)	0	$-(n^3+2n^2-n-2)$	$-\frac{1}{2}(n^3+3n^2+2n)$
(2-2) _a	$-(n^3+n^2)$	$-(2n^3+n^2)$	$-\frac{1}{2}(n^3+2n^2-n-2)$	$-(n^3+n^2)$	$-\frac{1}{2}(n^3+2n^2-n-2)$	$-\frac{1}{4}(n^3+3n^2+2n)$
(2-2) _b	$-2(n^2+n)$	$-2(2n^3+n^2)$	0	0	$-2(n^3+2n^2-n-2)$	$-(n^3+3n^2+2n)$
(2-2) _c	$-(n^3-n)$	0	$-\frac{1}{2}(n^3+2n^2-n-2)$	0	$\frac{1}{2}(n^3+2n^2-n-2)$	$\frac{1}{4}(n^3+3n^2+2n)$
(2-3) _a	0	$-(2n^2+n)$	0	0	$-(n^2+3n+2)$	$-\frac{1}{2}(n^2+3n+2)$
(2-3) _b	$\frac{1}{2}(n^2+n)$	0	$\frac{1}{4}(n^2+3n+2)$	0	$-\frac{1}{4}(n^2+3n+2)$	$-\frac{1}{8}(n^2+3n+2)$
(2-3) _c	$-\frac{1}{2}(n^2+n)$	$-(n^2+\frac{1}{2}n)$	$-\frac{1}{4}(n^2+3n+2)$	$-\frac{1}{2}(n^2+n)$	$-\frac{1}{4}(n^2+3n+2)$	$-\frac{1}{8}(n^2+3n+2)$
(3-1)	$(3n^2-\frac{9}{2}n)$	$-\frac{3}{2}n$	$\frac{3}{2}(n^2-1)$	$\frac{3}{2}n$	$-\frac{3}{2}(n^2-1)$	$-\frac{3}{4}(n^2+n)$
(3-2) _a	$\frac{1}{4}(2n^2+n)$	$(n^2+\frac{1}{4}n)$	$\frac{1}{4}(n^2-1)$	$\frac{1}{4}(2n^2+n)$	$\frac{1}{4}(n^2-1)$	$\frac{1}{8}(n^2+n)$
(3-2) _b	$-(\frac{1}{2}n^2-\frac{3}{4}n)$	$\frac{1}{4}n$	$-\frac{1}{4}(n^2-1)$	$-\frac{1}{4}n$	$\frac{1}{4}(n^2-1)$	$\frac{1}{8}(n^2+n)$
(3-2) _c	$2n$	$(2n^2+n)$	0	0	(n^2-1)	$\frac{1}{2}(n^2+n)$
(3-3) _a	$-(n+\frac{1}{2})$	$(n+\frac{1}{2})$	0	0	$\frac{1}{2}(n+1)$	$\frac{1}{4}(n+1)$
(3-3) _b	$-\frac{1}{8}$	$\frac{1}{2}n+\frac{1}{8}$	$\frac{1}{8}(n+1)$	$\frac{1}{8}(2n+1)$	$\frac{1}{8}(n+1)$	$\frac{1}{16}(n+1)$
(3-3) _c	$\frac{5}{2}n+\frac{5}{8}$	$-\frac{5}{8}$	$\frac{5}{8}(n+1)$	$\frac{5}{8}$	$-\frac{5}{8}(n+1)$	$-\frac{5}{16}(n+1)$
Total _{sub}	$-3(2n^3+n^2)$	$-3(3n^3+n^2)$	$(-3n^3-6n^2-\frac{3}{4}n+\frac{9}{4})$	$(-n^3-n^2+\frac{5}{4}n+\frac{3}{4})$	0	0

Table 18: $i\Delta_{\text{cf}} \times i[S]_{n \geq 1} - \text{II}'$. All contributions are multiplied by $\frac{\kappa^2 m H}{64\pi^4} \sum_{n \geq 1}^\infty \left(\frac{y}{4}\right)^n \ln\left(\frac{y}{4}\right)$.

Here Y stands for $\frac{y}{4}$. After the summation, the total contribution from $i\Delta_{\text{cf}} i[S]_{n \geq 1}$ is,

$$\begin{aligned}
-i[\Sigma^{\text{cfn1}}](x; x') = & \frac{\kappa^2 m H}{64\pi^4} \left\{ 8 \frac{\gamma^0 \gamma^k \Delta x_k}{\Delta x^6} \left[\frac{(-Y^2 + 7Y)}{(1-Y)^3} + \frac{(Y^3 + 6Y^2 - Y)}{(1-Y)^4} \ln(Y) \right] \right. \\
& + \left[5 \frac{Ha\gamma^0 \Delta \eta \gamma^k \Delta x_k}{\Delta x^6} + \frac{Ha' \gamma^0 \Delta \eta \gamma^k \Delta x_k}{\Delta x^6} - 3 \frac{Ha \Delta \eta^2}{\Delta x^6} - 3 \frac{Ha' \Delta \eta^2}{\Delta x^6} \right] \left[\frac{-3Y^3 + 26Y^2 + Y}{2(1-Y)^3} \right. \\
& + \left. \frac{3Y^3 + 22Y^2 - 3Y}{2(1-Y)^4} \ln(Y) \right] + \left[\frac{Ha}{\Delta x^4} + \frac{Ha'}{\Delta x^4} \right] \left[\frac{-12(Y^2 + 2Y)}{(1-Y)^3} + \frac{-3(Y^3 + 8Y^2 + 3Y)}{(1-Y)^4} \ln(Y) \right] \\
& + \frac{H^2 a^2 \gamma^0 \gamma^k \Delta x_k}{\Delta x^4} \left[\frac{-3(Y^2 - 6Y + 29)}{4(1-Y)^3} + \frac{-3(3Y^4 - 12Y^3 + 23Y^2 + 10Y)}{4(1-Y)^4} \ln(Y) \right] \\
& \left. + \frac{H^2 a a' \gamma^0 \gamma^k \Delta x_k}{\Delta x^4} \left[\frac{(5Y^3 - 14Y^2 - 15Y)}{4(1-Y)^3} + \frac{(-3Y^4 + 14Y^3 - 35Y^2)}{4(1-Y)^4} \ln(Y) \right] \right\}. \quad (144)
\end{aligned}$$

5.2 Sub-Leading Contributions from $i\delta\Delta_A$

In this subsection we work out the contribution from substituting the residual A -type part of the graviton propagator in Table 5,

$$i[\alpha\beta\Delta_{\rho\sigma}](x; x') \longrightarrow [\bar{\eta}_{\alpha\rho}\bar{\eta}_{\sigma\beta} + \bar{\eta}_{\alpha\sigma}\bar{\eta}_{\rho\beta} - \frac{2}{D-3}\bar{\eta}_{\alpha\beta}\bar{\eta}_{\rho\sigma}] i\delta\Delta_A(x; x'). \quad (145)$$

As with the conformal contributions of the previous section we first make the requisite contractions and then act the derivatives. The result of this first step is summarized in Table 19 and Table 20. We have sometimes broken the result for a single vertex pair into as many as five terms because the three different tensors in (145) can make distinct contributions, and because distinct contributions also come from breaking up factors of $\gamma^\alpha J^{\beta\mu}$. These distinct contributions are labeled by subscripts a , b , c , etc.

The next step is to act the derivatives and it is of course necessary to have an expression for $i\delta\Delta_A(x; x')$. From (66) one can infer,

$$\begin{aligned}
i\delta\Delta_A(x; x') = & \frac{H^2}{16\pi^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2}+1)}{\frac{D}{2}-2} \frac{(aa')^{2-\frac{D}{2}}}{\Delta x^{D-4}} + \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \left\{ -\pi \cot\left(\frac{\pi}{2}D\right) + \ln(aa') \right\} \\
& + \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \sum_{n=1}^{\infty} \left\{ \frac{1}{n} \frac{\Gamma(n+D-1)}{\Gamma(n+\frac{D}{2})} \left(\frac{y}{4}\right)^n - \frac{1}{n-\frac{D}{2}+2} \frac{\Gamma(n+\frac{D}{2}+1)}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right\}. \quad (146)
\end{aligned}$$

I	J	sub	$iV_I^{\alpha\beta}(x) i[S](x; x') iV_J^{\rho\sigma}(x') [\alpha_\beta T_{\rho\sigma}^A] i\delta\Delta_A(x; x')$
1	4		$-\frac{(D-1)}{(D-3)} i\kappa^2 am \bar{\theta} i[S](x; x') \delta\Delta_A(x; x')$
2	4		$\frac{1}{(D-3)} i\kappa^2 am \bar{\theta} i[S](x; x') i\delta\Delta_A(x; x')$
3	4	a	$-\frac{(D-1)}{2(D-3)} i\kappa^2 am \gamma^0 \partial_0 i\delta\Delta_A(x; x') i[S](x; x')$
3	4	b	$-\frac{(D-2)}{2(D-3)} i\kappa^2 am \bar{\theta} i\delta\Delta_A(x; x') i[S](x; x')$
4	1		$\frac{(D-1)}{(D-3)} i\kappa^2 am \partial'_\mu \{i[S](x; x') \gamma^\mu i\delta\Delta_A(x; x')\}$
4	2		$\frac{1}{(D-3)} i\kappa^2 am \partial_k \{i[S](x; x') \gamma_k i\delta\Delta_A(x; x')\}$
4	3	a	$-\frac{(D-1)}{2(D-3)} i\kappa^2 am i[S](x; x') \gamma^0 \partial_0 i\delta\Delta_A(x; x')$
4	3	b	$-\frac{(D-2)}{2(D-3)} i\kappa^2 am i[S](x; x') \bar{\theta} i\delta\Delta_A(x; x')$

Table 19: Contractions from the $i\delta\Delta_A$ part of the graviton propagator-I

In $D=4$ the most singular contributions to (99) have the form, $i\delta\Delta_A/\Delta x^4$. Because the infinite series terms in (146) go like positive powers of Δx^2 these terms make integrable contributions to the quantum-corrected Dirac equation (1). We can therefore take $D=4$ for those terms, at which point all the infinite series terms drop. Hence it is only necessary to keep the first line of (146) and that is all we shall ever use.

The generic contraction from Table 19 which only consists of one derivative acting on a propagator, the order m contributions must be and could only be from the most singular part of the fermion propagator. In reducing these terms the following derivatives occur many times,

$$\partial_i i\delta\Delta_A(x; x') = -\frac{H^2}{8\pi^{\frac{D}{2}}} \Gamma\left(\frac{D}{2}+1\right) (aa')^{2-\frac{D}{2}} \frac{\Delta x^i}{\Delta x^{D-2}} = -\partial'_i i\delta\Delta_A(x; x'), \quad (147)$$

$$\begin{aligned} \partial_0 i\delta\Delta_A(x; x') = \frac{H^2}{8\pi^{\frac{D}{2}}} \Gamma\left(\frac{D}{2}+1\right) (aa')^{2-\frac{D}{2}} & \left\{ \frac{\Delta\eta}{\Delta x^{D-2}} - \frac{aH}{2\Delta x^{D-4}} \right\} \\ & + \frac{H^{D-2}}{2^D \pi^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} aH, \quad (148) \end{aligned}$$

$$\partial'_0 i\delta\Delta_A(x; x') = \frac{H^2}{8\pi^{\frac{D}{2}}} \Gamma\left(\frac{D}{2}+1\right) (aa')^{2-\frac{D}{2}} \left\{ -\frac{\Delta\eta}{\Delta x^{D-2}} - \frac{a'H}{2\Delta x^{D-4}} \right\}$$

I	J	sub	$iV_I^{\alpha\beta}(x) i[S](x; x') iV_f^{\rho\sigma}(x') [\alpha_\beta T_{\rho\sigma}^A] i\delta\Delta_A(x; x')$
1	1		$\frac{(D-1)}{(D-3)} \kappa^2 \partial'_\mu \{ \not{\partial} i[S](x; x') \gamma^\mu i\delta\Delta_A(x; x') \}$
1	2		$\frac{1}{(D-3)} \kappa^2 \partial_k \{ \not{\partial} i[S](x; x') \gamma_k i\delta\Delta_A(x; x') \}$
1	3	a	$-\frac{(D-1)}{2(D-3)} \kappa^2 \not{\partial} i[S](x; x') \gamma^0 \partial'_0 i\delta\Delta_A(x; x')$
1	3	b	$\frac{(D-2)}{2(D-3)} \kappa^2 \not{\partial} i[S](x; x') \bar{\not{\partial}} i\delta\Delta_A(x; x')$
2	1		$-\frac{1}{(D-3)} \kappa^2 \partial'_\mu \{ \bar{\not{\partial}} i[S](x; x') \gamma^\mu i\delta\Delta_A(x; x') \}$
2	2	a	$\frac{1}{4} \kappa^2 \bar{\not{\partial}} \{ \partial_k i[S](x; x') \gamma_k i\delta\Delta_A(x; x') \}$
2	2	b	$+\frac{1}{4} \kappa^2 \partial_\ell \{ \gamma_k \partial_\ell i[S](x; x') \gamma_k i\delta\Delta_A(x; x') \}$
2	2	c	$-\frac{1}{2(D-3)} \kappa^2 \partial_k \{ \bar{\not{\partial}} i[S](x; x') \gamma_k i\delta\Delta_A(x; x') \}$
2	3	a	$\frac{1}{2(D-3)} \kappa^2 \bar{\not{\partial}} i[S](x; x') \gamma^0 \partial'_0 i\delta\Delta_A(x; x')$
2	3	b	$-\frac{1}{4} \kappa^2 \gamma_k \partial_\ell i[S](x; x') \gamma_{(k} \partial_{\ell)} i\delta\Delta_A(x; x')$
2	3	c	$-\frac{1}{4(D-3)} \kappa^2 \bar{\not{\partial}} i[S](x; x') \bar{\not{\partial}} i\delta\Delta_A(x; x')$
3	1	a	$\frac{1}{2} \left(\frac{D-1}{D-3} \right) \kappa^2 \partial'_\mu \{ \gamma^0 \partial_0 i\delta\Delta_A(x; x') i[S](x; x') \gamma^\mu \}$
3	1	b	$\frac{(D-2)}{2(D-3)} \kappa^2 \partial'_\mu \{ \bar{\not{\partial}} i\delta\Delta_A(x; x') i[S](x; x') \gamma^\mu \}$
3	2	a	$\frac{1}{2(D-3)} \kappa^2 \partial_k \{ \gamma^0 \partial_0 i\delta\Delta_A(x; x') i[S](x; x') \gamma_k \}$
3	2	b	$\frac{1}{4(D-3)} \kappa^2 \partial_k \{ \bar{\not{\partial}} i\delta\Delta_A(x; x') i[S](x; x') \gamma_k \}$
3	2	c	$\frac{1}{8} \kappa^2 \bar{\not{\partial}} \{ i[S](x; x') \bar{\not{\partial}} i\delta\Delta_A(x; x') \}$
3	2	d	$\frac{1}{8} \kappa^2 \partial_k \{ \gamma_\ell i[S](x; x') \gamma_\ell \partial_k i\delta\Delta_A(x; x') \}$
3	3	a	$-\frac{1}{4} \left(\frac{D-1}{D-3} \right) \kappa^2 \gamma^0 i[S](x; x') \gamma^0 \partial_0 \partial'_0 i\delta\Delta_A(x; x')$
3	3	b	$\frac{(D-2)}{4(D-3)} \kappa^2 \gamma^0 i[S](x; x') \partial_0 \bar{\not{\partial}} i\delta\Delta_A(x; x')$
3	3	c	$-\frac{(D-2)}{4(D-3)} \kappa^2 \gamma_k i[S](x; x') \partial_k \gamma^0 \partial'_0 i\delta\Delta_A(x; x')$
3	3	d	$\frac{3D-7}{16(D-3)} \kappa^2 \gamma_k i[S](x; x') \partial_k \bar{\not{\partial}} i\delta\Delta_A(x; x')$
3	3	e	$-\frac{1}{16} \kappa^2 \gamma_k i[S](x; x') \gamma_k \nabla^2 i\delta\Delta_A(x; x')$

Table 20: Contractions from the $i\delta\Delta_A$ part of the graviton propagator-II

$(I - J)_{\text{sub}}$	$\frac{i\kappa^2 H^2}{16\pi^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2}+1)}{2} \frac{ma}{(D-4)} \delta^D(x - x')$	$\frac{i\kappa^2 H^2}{16\pi^2} ma \delta^4(x - x')$
$(1 - 4)$	$\frac{32}{\sqrt{\pi}} \frac{H^{D-4}}{(D-3)}$	$-12 \ln(a)$
$(2 - 4)$	$-\mu^{D-4} \frac{(D-1)}{(D-3)^2}$	$3 \ln(a)$
$(3 - 4)_{\text{a}}$	$-\mu^{D-4} \frac{(D-1)}{2(D-3)^2}$	$\frac{3}{2} \ln(a)$
$(3 - 4)_{\text{b}}$	$-\mu^{D-4} \frac{(D-1)(D-2)}{2(D-3)^2}$	$3 \ln(a)$
$(4 - 1)$	0	$9 + 6 \ln(\frac{H^2}{4\mu^2})$
$(4 - 2)$	0	0
$(4 - 3)_{\text{a}}$	$-\mu^{D-4} \frac{(D-1)}{2(D-3)^2}$	$\frac{3}{2} \ln(a)$
$(4 - 3)_{\text{b}}$	$-\mu^{D-4} \frac{(D-1)(D-2)}{2(D-3)^2}$	$3 \ln(a)$
total	$\frac{32}{\sqrt{\pi}} \frac{H^{D-4}}{(D-3)} - \mu^{D-4} \frac{D(D-1)}{(D-3)^2}$	$9 + 6 \ln(\frac{H^2}{4\mu^2})$

Table 21: The local terms from $i\delta\Delta_A \times i[S]_{\text{cf}}$.

$$+ \frac{H^{D-2}}{2^D \pi^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} a' H . \quad (149)$$

We also make use of following identities to simplify the contributions,

$$\frac{\Delta\eta^2}{\Delta x^{2D-2}} = \frac{1}{4(D-2)(D-3)} \partial_0^2 \frac{1}{\Delta x^{2D-6}} - \frac{1}{2(D-2)} \frac{1}{\Delta x^{2D-4}} , \quad (150)$$

$$\frac{\overline{\Delta x}^2}{\Delta x^{2D-2}} = \frac{1}{4(D-2)(D-3)} \nabla^2 \frac{1}{\Delta x^{2D-6}} + \frac{(D-1)}{2(D-2)} \frac{1}{\Delta x^{2D-4}} , \quad (151)$$

$$\frac{\gamma^0 \Delta\eta \gamma^k \Delta x_k}{\Delta x^{2D-2}} = \frac{-1}{4(D-2)(D-3)} \gamma^0 \partial_0 \bar{\partial} \frac{1}{\Delta x^{2D-6}} , \quad (152)$$

$$\frac{\Delta\eta}{\Delta x^{2D-4}} = \frac{1}{2(D-3)} \partial_0 \frac{1}{\Delta x^{2D-6}} , \quad \frac{\gamma^k \Delta x_k}{\Delta x^{2D-4}} = \frac{-1}{2(D-3)} \bar{\partial} \frac{1}{\Delta x^{2D-6}} . \quad (153)$$

Note that the contraction (1-1) produces a delta function through (103) and picks up one divergent and one finite local term which do not possess any

dimension-dependent powers of Δx . We tabulate this kind of the result in Table 21. One might already notice that the first two terms of (146)¹² and the final two terms of (148) and (149) tend to cancel in $D = 4$. We indeed encounter this kind of cancelation entirely for the leading divergent terms in contractions (4-1) and (4-2) and hence we present the remaining finite results in Table 22. For the rest of the contractions, the sum of the leading divergent contributions do not vanish after we apply (150), (151), (152), (153), (127), (128), (129), (110) and (111). Therefore we give the local terms in Table 21 and the finite nonlocal terms in Table 22. After collecting all contributions from Table 21 and Table 22 we obtain,

$$\begin{aligned}
-i[\Sigma^{\text{idAcf}}](x;x') &= \frac{i\kappa^2 H^2}{16\pi^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2}+1)}{2} \frac{ma}{(D-4)} \left[\frac{32}{\sqrt{\pi}} \frac{H^{D-4}}{(D-3)} - \frac{\mu^{D-4} D(D-1)}{(D-3)^2} \right] \\
&\times \delta^D(x-x') + \frac{i\kappa^2 H^2}{16\pi^2} \left[9 + 6 \ln\left(\frac{H^2}{4\mu^2}\right) \right] ma \delta^4(x-x') + \frac{\kappa^2 H^2}{32\pi^4} ma \left\{ [6\partial^2 - 2\nabla^2] \right. \\
&\times \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] + \left[\frac{3}{2} \partial_0^2 - \frac{1}{2} \gamma^0 \partial_0 \bar{\partial} - \left(\frac{9}{2} + 2 \ln\left(\frac{H^2}{4\mu^2}\right) \right) \nabla^2 \right] \frac{1}{\Delta x^2} \left. \right\}. \quad (154)
\end{aligned}$$

The generic contractions from Table 20, which are comprised of two derivatives acting on the propagators, the order m contributions could either come from the flat spacetime mass term or from the infinite series expansion of the fermion propagator.

We first deal with the contributions from the flat spacetime mass term of the fermion propagator. At $D = 4$ these terms have a dimension $\frac{i\delta\Delta_A}{\Delta x^4}$ which are not integrable when we substitute back to the quantum-corrected Dirac equation and working on an arbitrary D is required. The contractions (1-1), (1-2), (2-1) and (2-2) tend to cancel through (146). It turns out that the sum of divergent terms vanish and only left the finite terms. Hence we present the results for $D = 4$ at the end. Here we showed (2-1) as an example,

$$\begin{aligned}
&\frac{-\kappa^2}{(D-3)} \partial'_\mu \left\{ \bar{\partial} i[S] \gamma^\mu i\delta\Delta_A \right\} = \frac{\kappa^2 H^2}{32\pi^D} \frac{\Gamma(\frac{D}{2})\Gamma(\frac{D}{2}+1)}{(D-3)} ma \partial'_\mu \left\{ \frac{(aa')^{2-\frac{D}{2}}}{\frac{1}{2}(D-4)} \right. \\
&\left. \frac{\gamma^k \Delta x_k \gamma^\mu}{\Delta x^{2D-4}} \right\} + \frac{\kappa^2 H^{D-2}}{2^{D+1}\pi^D} \frac{\Gamma(D-1)}{(D-3)} ma \partial'_\mu \left\{ \left[\frac{-2}{(D-4)} + \ln(aa') \right] \frac{\gamma^k \Delta x_k \gamma^\mu}{\Delta x^D} \right\} \\
&= \frac{\kappa^2 H^2}{32\pi^D} \frac{\mu^{D-4} ma}{(D-3)} \left\{ \left[\frac{2}{(D-4)} - \frac{2}{(D-4)} - \frac{1}{2} \ln\frac{H^2}{4\mu^2} - 1 \right] \bar{\partial} \bar{\partial} \frac{1}{\Delta x^2} - \bar{\partial} \bar{\partial} \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right\}
\end{aligned}$$

¹² $\pi \cot\left(\frac{D\pi}{2}\right) = \frac{1}{\frac{D}{2}(D-4)}$

$(I-J)_{\text{sub}}$	$\partial^2 \frac{\ln(\mu\Delta x)^2}{\Delta x^2}$	$\gamma^0 \partial_0 \bar{\phi} \frac{\ln(\mu\Delta x)^2}{\Delta x^2}$	$\nabla^2 \frac{\ln(\mu\Delta x)^2}{\Delta x^2}$	$\partial_0^2 \frac{1}{\Delta x^2}$	$\gamma^0 \partial_0 \bar{\phi} \frac{1}{\Delta x^2}$	$\nabla^2 \frac{1}{\Delta x^2}$
(1-4)	0	0	0	0	0	0
(2-4)	$\frac{3}{4}$	-1	-1	0	$-2 - \ln(\frac{H^2}{4\mu^2})$	$-2 - \ln(\frac{H^2}{4\mu^2})$
(3-4) _a	$\frac{3}{8}$	0	0	$\frac{3}{4}$	$\frac{3}{4}$	0
(3-4) _b	$\frac{3}{4}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$
(4-1)	3	0	0	0	0	0
(4-2)	0	1	-1	0	$\frac{3}{2} + \ln(\frac{H^2}{4\mu^2})$	$-\frac{3}{2} - \ln(\frac{H^2}{4\mu^2})$
(4-3) _a	$\frac{3}{8}$	0	0	$\frac{3}{4}$	$-\frac{3}{4}$	0
(4-3) _b	$\frac{3}{4}$	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
total	6	0	-2	$\frac{3}{2}$	$-\frac{1}{2}$	$-\frac{9}{2} - 2 \ln(\frac{H^2}{4\mu^2})$

Table 22: The non-local terms from $i\delta\Delta_A \times i[S]_{\text{cf}}$. All contributions are multiplied by $\frac{i\kappa^2 H^2}{32\pi^4} ma$.

$(I-J)$	$\partial^2 \frac{\ln(\mu\Delta x)^2}{\Delta x^2}$	$\gamma^0 \partial_0 \bar{\phi} \frac{\ln(\mu\Delta x)^2}{\Delta x^2}$	$\nabla^2 \frac{\ln(\mu\Delta x)^2}{\Delta x^2}$	$Ha \partial_0 \frac{\ln(\mu\Delta x)^2}{\Delta x^2}$	$Ha \gamma^0 \bar{\phi} \frac{\ln(\mu\Delta x)^2}{\Delta x^2}$
(1-1)	-3	0	0	3	3
(1-2)	0	-1	1	0	-1
(2-1)	0	1	1	0	0
(2-2)	0	0	$\frac{1}{2}$	0	0
total	-3	0	$\frac{5}{2}$	3	2

Table 23: $i\delta\Delta_A \times i[S]_{\text{fm}}$. All contributions are multiplied by $\frac{\kappa^2 H^2}{32\pi^4} ma$.

$$= \frac{\kappa^2 H^2}{32\pi^4} ma \left\{ \left[-\frac{3}{2} - \ln \frac{H^2}{4\mu^2} \right] \bar{\partial} \partial \frac{1}{\Delta x^2} - \bar{\partial} \partial \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right\}. \quad (155)$$

The most singular terms in the contractions (2-3)_a, (3-1) and (3-2) are integrable after exacting out derivatives¹³. Because the contraction (3-3) involves various double derivatives directly acting on $i\delta\Delta_A$ we would give their expressions here,

$$\begin{aligned} \partial_0 \partial'_0 i\delta\Delta_A = & \frac{H^2 \Gamma(\frac{D}{2}+1)}{8\pi^{\frac{D}{2}} (aa')^{\frac{D}{2}-2}} \left\{ \frac{-1}{\Delta x^{D-2}} - \frac{(D-2)\Delta\eta^2}{\Delta x^D} + \frac{1}{2}(D-4) \right. \\ & \left. \times \left[\frac{H^2 aa' \Delta\eta^2}{\Delta x^{D-2}} + \frac{1}{2} \frac{H^2 aa'}{\Delta x^{D-4}} \right] \right\}, \quad (156) \end{aligned}$$

$$\partial_k \partial_0 i\delta\Delta_A = \frac{H^2 \Gamma(\frac{D}{2}+1)}{8\pi^{\frac{D}{2}} (aa')^{\frac{D}{2}-2}} \left\{ \frac{-(D-2)\Delta\eta\Delta x_k}{\Delta x^D} + \frac{1}{2}(D-4) \frac{Ha\Delta x_k}{\Delta x^{D-2}} \right\}, \quad (157)$$

$$\partial_k \partial'_0 i\delta\Delta_A = \frac{H^2 \Gamma(\frac{D}{2}+1)}{8\pi^{\frac{D}{2}} (aa')^{\frac{D}{2}-2}} \left\{ \frac{(D-2)\Delta\eta\Delta x_k}{\Delta x^D} + \frac{1}{2}(D-4) \frac{Ha'\Delta x_k}{\Delta x^{D-2}} \right\}, \quad (158)$$

$$\partial_k \partial_l i\delta\Delta_A = \frac{H^2 \Gamma(\frac{D}{2}+1)}{8\pi^{\frac{D}{2}} (aa')^{\frac{D}{2}-2}} \left\{ \frac{(D-2)\Delta x_k \Delta x_l}{\Delta x^D} - \frac{\delta_{kl}}{\Delta x^{D-2}} \right\}. \quad (159)$$

Note that the terms with a factor of $(D-4)$ in the equations (156)-(158) have dimensionality, which is either $\frac{1}{\Delta x^{D-4}}$ or $\frac{1}{\Delta x^{D-3}}$. When they combine with $i[S]_{\text{fm}}$ whose dimensionality is $\frac{1}{\Delta x^{D-2}}$, one can see that those contributions are integrable in four dimensions and hence could only give the contributions of the order $(D-4)$. Therefore we can drop the terms we mentioned above from (156) to (158) when we compute the contraction (3-3). In addition, the contractions (3-3)_b and (3-3)_c are both finite in $D=4$ dimensions after performing the partial integration¹⁴. The finite contributions mentioned above are displayed in Table 23 and Table 24 and they can be read off immediately,

$$\begin{aligned} & \frac{\kappa^2 H^2 ma}{32\pi^4} \left\{ \left[-3\partial^2 + \frac{5}{2}\nabla^2 + 3Ha\partial_0 + 2Ha\gamma^0 \bar{\partial} \right] \frac{\ln \mu^2 \Delta x^2}{\Delta x^2} + \left[-3 \ln \frac{H^2}{4\mu^2} \partial^2 - \frac{7}{4} \gamma^0 \partial_0 \bar{\partial} \right. \right. \\ & \left. \left. + \left(\frac{1}{2} + \frac{5}{2} \ln \frac{H^2}{4\mu^2} \right) \nabla^2 + \left(\frac{3}{2} + 3 \ln \frac{H^2}{4\mu^2} \right) Ha\partial_0 + \left(1 + 2 \ln \frac{H^2}{4\mu^2} \right) Ha\gamma^0 \bar{\partial} \right] \frac{1}{\Delta x^2} \right\}. \quad (160) \end{aligned}$$

¹³Some less singular terms in the contractions (2-3)_a, (3-1)_a and (3-2)_a get canceled through the property of (148) and (149).

¹⁴“Partial integration” is not standard usage in Physics. Here we mean extracting the derivatives outside the quantum-corrected Dirac equation.

$(I - J)_{\text{sub}}$	$\partial^2 \frac{1}{\Delta x^2}$	$\gamma^0 \partial_0 \bar{\partial} \frac{1}{\Delta x^2}$	$\nabla^2 \frac{1}{\Delta x^2}$	$Ha \partial_0 \frac{1}{\Delta x^2}$	$Ha \gamma^0 \bar{\partial} \frac{1}{\Delta x^2}$
$(1 - 1)$	$-\frac{3}{2} - 3 \ln \frac{H^2}{4\mu^2}$	0	0	$\frac{3}{2} + 3 \ln \frac{H^2}{4\mu^2}$	$\frac{3}{2} + 3 \ln \frac{H^2}{4\mu^2}$
$(1 - 2)$	0	$-\frac{3}{2} - \ln \frac{H^2}{4\mu^2}$	$\frac{3}{2} + \ln \frac{H^2}{4\mu^2}$	0	$-\frac{1}{2} - \ln \frac{H^2}{4\mu^2}$
$(2 - 1)$	0	$\frac{3}{2} + \ln \frac{H^2}{4\mu^2}$	$\frac{3}{2} + \ln \frac{H^2}{4\mu^2}$	0	0
$(2 - 2)$	0	0	$\frac{3}{4} + \frac{1}{2} \ln \frac{H^2}{4\mu^2}$	0	0
$(2 - 3)_a$	0	$\frac{1}{4}$	0	0	0
$(3 - 1)_a$	$\frac{3}{2}$	$-\frac{3}{2}$	$-\frac{3}{2}$	0	0
$(3 - 1)_b$	0	-1	-1	0	0
$(3 - 2)$	0	$\frac{1}{2}$	$-\frac{3}{4}$	0	0
$(3 - 3)_{b+c}$	0	$\frac{1}{4} - \frac{1}{4}$	0	0	0
total	$-3 \ln \frac{H^2}{4\mu^2}$	$-\frac{7}{4}$	$\frac{1}{2} + \frac{5}{2} \ln \frac{H^2}{4\mu^2}$	$\frac{3}{2} + 3 \ln \frac{H^2}{4\mu^2}$	$1 + 2 \ln \frac{H^2}{4\mu^2}$

Table 24: $i\delta\Delta_A \times i[S]_{\text{fm}}$. All contributions are multiplied by $\frac{\kappa^2 H^2}{32\pi^4} ma$.

(I-J) _{sub}	$\frac{\Delta\eta^2}{\Delta x^{2D-2}}$	$\frac{\overline{\Delta x}^2}{\Delta x^{2D-2}}$	$\frac{1}{\Delta x^{2D-4}}$	$\frac{\gamma^0\Delta\eta\gamma^k\Delta x_k}{\Delta x^{2D-2}}$	$\frac{Ha\Delta\eta}{\Delta x^{2D-4}}$	$\frac{Ha\gamma^0\gamma^k\Delta x_k}{\Delta x^{2D-4}}$
(1-3) _a	$-(D-1)$	0	0	$(D-1)$	$\frac{(D-1)}{(D-2)}$	0
(1-3) _b	0	$\frac{-(D-2)^2}{(D-3)}$	0	$\frac{-(D-2)^2}{(D-3)}$	0	$\frac{-(D-2)}{(D-3)}$
(2-3) _{b+c}	0	$\frac{(D-1)(D-2)^2}{4(D-3)}$	0	0	0	0
(3-3) _a	$\frac{(D-1)(D-2)}{2(D-3)}$	0	$\frac{(D-1)}{2(D-3)}$	0	0	0
(3-3) _{d+e}	0	$\frac{(D-2)^2(D-5)}{8(D-3)}$	$\frac{-(D-1)(D-2)(D-5)}{8(D-3)}$	0	0	0
total	$\frac{(D-1)(3D-8)}{2(D-3)}$	$\frac{3(D-2)^2(D-5)}{8(D-3)}$	$\frac{-(D-1)^2(D-6)}{8(D-3)}$	$\frac{-1}{(D-3)}$	$\frac{(D-1)}{(D-2)}$	$\frac{-(D-2)}{(D-3)}$

Table 25: The contribution from $i\delta\Delta_A \times i[S]_{\text{fm}}$ which are not integrable in four dimensions. Note that all contributions are multiplied by the factor $\frac{\kappa^2 H^2}{64\pi^D} \Gamma(\frac{D}{2}-1) \Gamma(\frac{D}{2}+1) ma(aa')^{2-\frac{D}{2}}$.

The rest of the contractions which require further renormalization are summarized in Table 25. We then apply the same formalism to partially integrate, extract the local divergences and take $D = 4$ for the remaining, integrable and ultraviolet finite nonlocal terms. The sub-total from Table 25 could be obtained,

$$\begin{aligned}
& \frac{i\kappa^2 H^2}{64\pi^{\frac{D}{2}}} \Gamma(\frac{D}{2}+1) \frac{\mu^{D-4}(D-1)(D^3-9D^2+20D-4)}{8(D-2)(D-3)^2(D-4)} ma\delta^D(x-x') \\
& + \frac{i\kappa^2 H^2}{64\pi^2} \frac{3ma}{2} \ln a\delta^4(x-x') + \frac{\kappa^2 H^2 ma}{32\pi^4} \left\{ \frac{3}{32} \partial^2 \frac{\ln \mu^2 \Delta x^2}{\Delta x^2} \right. \\
& \quad \left. + \left[\frac{3}{4} \partial_0^2 + \frac{1}{8} \gamma^0 \partial_0 \bar{\partial} - \frac{3}{16} \nabla^2 + \frac{3}{4} Ha \partial_0 + Ha \gamma^0 \bar{\partial} \right] \frac{1}{\Delta x^2} \right\}. \quad (161)
\end{aligned}$$

Combining (160) and (161) gives,

$$\begin{aligned}
& -i[\Sigma^{\text{idAfm}}](x; x') = \frac{i\kappa^2 H^2}{64\pi^{\frac{D}{2}}} \Gamma(\frac{D}{2}+1) \frac{\mu^{D-4}(D-1)(D^3-9D^2+20D-4)}{8(D-2)(D-3)^2(D-4)} \times \\
& ma\delta^D(x-x') + \frac{i\kappa^2 H^2}{64\pi^2} \frac{3ma}{2} \ln a\delta^4(x-x') + \frac{\kappa^2 H^2 ma}{32\pi^4} \left\{ \left[-\frac{93}{32} \partial^2 + \frac{5}{2} \nabla^2 + 3Ha \partial_0 \right. \right.
\end{aligned}$$

I-J _{sub}		$\frac{\Delta\eta^3}{\Delta x^6}$	$\frac{\gamma^0\Delta\eta^2\gamma^k\Delta x_k}{\Delta x^6}$	$\frac{\Delta\eta}{\Delta x^4}$	$\frac{\gamma^0\gamma^k\Delta x_k}{\Delta x^4}$
(1-1)	1	0	0	$24(3n^2+4n+1)$	$-24(3n^2+4n+1)$
(1-1)	$\ln Y$	0	0	$24(n^3+2n^2+n)$	$-24(n^3+2n^2+n)$
(1-2)	1	0	0	0	$8(3n^2-1)$
(1-2)	$\ln Y$	0	0	0	$8(n^3-n)$
(2-1)	1	$-16(3n^2-6n+2)$	$16(3n^2-6n+2)$	$-8(6n^2-6n+1)$	$8(3n^2-2n)$
(2-1)	$\ln Y$	$-16(n^3-3n^2+2n)$	$16(n^3-3n^2+2n)$	$-8(2n^3-3n^2+n)$	$8(n^3-n^2)$
(2-2) _a	1	$-2(3n^2-6n+2)$	$2(3n^2-6n+2)$	$-(6n^2-6n+1)$	$(6n^2-2n-1)$
(2-2) _a	$\ln Y$	$-2(n^3-3n^2+2n)$	$2(n^3-3n^2+2n)$	$-(2n^3-3n^2+n)$	$(2n^3-n^2-n)$
(2-2) _b	1	$-6(3n^2-6n+2)$	$-2(3n^2-6n+2)$	$-3(6n^2-6n+1)$	$-(6n^2-2n-1)$
(2-2) _b	$\ln Y$	$-6(n^3-3n^2+2n)$	$-2(n^3-3n^2+2n)$	$-3(2n^3-3n^2+n)$	$-(2n^3-n^2-n)$
(2-2) _c	1	$4(3n^2-6n+2)$	$-4(3n^2-6n+2)$	$2(6n^2-6n+1)$	$-2(6n^2-2n-1)$
(2-2) _c	$\ln Y$	$-4(n^3-3n^2+2n)$	$-4(n^3-3n^2+2n)$	$2(2n^3-3n^2+n)$	$-2(2n^3-n^2-n)$
total	1	$-20(3n^2-6n+2)$	$12(3n^2-6n+2)$	$2(6n^2+78n+7)$	$-6(6n^2+18n+5)$
total	$\ln Y$	$-20(n^3-3n^2+2n)$	$12(n^3-3n^2+2n)$	$2(2n^3+39n^2+7n)$	$-6(2n^3+9n^2+5n)$

Table 26: $i\delta\Delta_A \times i[S]_{n \geq 0} - \text{IA}$. The factor $\frac{i\kappa^2 H^2}{2^6 \pi^4} \frac{mHaa'}{2} \left(\ln \frac{H^2 \Delta x^2}{4} \right) \sum_{n=0}^{\infty} Y^n$ multiplies all contributions. Here $Y = \frac{y}{4}$; $\ln Y$ and 1 are the multiplicative factors for the each individual row.

$$\begin{aligned}
& +2Ha\gamma^0\bar{\partial}\left[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\right] + \left[\left(\frac{3}{4}+3\ln\frac{H^2}{4\mu^2}\right)\partial_0 - \frac{13}{8}\gamma^0\partial_0\bar{\partial} + \left(\frac{5}{16}-\frac{1}{2}\ln\frac{H^2}{4\mu^2}\right)\nabla^2\right. \\
& \left. + \left(\frac{9}{4}+3\ln\frac{H^2}{4\mu^2}\right)Ha\partial_0 + \left(2+2\ln\frac{H^2}{4\mu^2}\right)Ha\gamma^0\bar{\partial}\right]\frac{1}{\Delta x^2}\Bigg\}. \quad (162)
\end{aligned}$$

The final class is comprised of terms in which comes from the infinite series expansion. Theses contributions are integrable in $D = 4$ and do not require any further renormalizations so we could set $D = 4$ right at the beginning without alternating the final result. Therefore we shall apply the infinite series expansion of the fermion propagator for $D = 4$ to this calculation. It is actually the same expression as (137). In addition, one can also use (138) and all related identities in Appendix A but remember that the series is summed up from $n = 0$ instead of $n = 1$. The residual A-type graviton propagator

I-J _{sub}		$\frac{Ha\Delta\eta^2}{\Delta x^4}$	$\frac{Ha'\Delta\eta^2}{\Delta x^4}$	$\frac{Ha\gamma^0\Delta\eta\gamma^k\Delta x_k}{\Delta x^4}$	$\frac{Ha'\gamma^0\Delta\eta\gamma^k\Delta x_k}{\Delta x^4}$	$\frac{Ha}{\Delta x^2}$
(1-1)	1	$24(3n^2+4n+1)$	0	$-24(3n^2+4n+1)$	0	$12(3n^2+6n+2)$
(1-1)	$\ln Y$	$24(n^3+2n^2+n)$	0	$-24(n^3+2n^2+n)$	0	$12(n^3+3n^2+2n)$
(1-2)	1	0	0	$8(3n^2-1)$	0	0
(1-2)	$\ln Y$	0	0	$8(n^3-n)$	0	0
(2-1)	1	$4(3n^2-1)$	$-4(3n^2-1)$	$-4(3n^2-1)$	$4(3n^2-1)$	$2(6n^2+6n+1)$
(2-1)	$\ln Y$	$4(n^3-n)$	$-4(n^3-n)$	$-4(n^3-n)$	$4(n^3-n)$	$2(2n^3+3n^2+n)$
(2-2) _a	1	$(3n^2-1)$	0	0	0	$\frac{1}{2}(6n^2+6n+1)$
(2-2) _a	$\ln Y$	(n^3-n)	0	0	0	$\frac{1}{2}(2n^3+3n^2+n)$
(2-2) _b	1	$3(3n^2-1)$	0	0	0	$\frac{3}{2}(6n^2+6n+1)$
(2-2) _b	$\ln Y$	$3(n^3-n)$	0	0	0	$\frac{3}{2}(2n^3+3n^2+n)$
(2-2) _c	1	$-2(3n^2-1)$	0	0	0	$-(6n^2+6n+1)$
(2-2) _c	$\ln Y$	$-2(n^3-n)$	0	0	0	$-(2n^3+3n^2+n)$
total	1	$6(15n^2+16n+3)$	$-4(3n^2-1)$	$-4(15n^2+24n+7)$	$4(3n^2-1)$	$9(6n^2+10n+3)$
total	$\ln Y$	$6(5n^3+8n^2+3n)$	$-4(n^3-n)$	$-4(5n^3+12n^2+7n)$	$4(n^3-n)$	$9(2n^3+5n^2+3n)$

Table 27: $i\delta\Delta_A \times i[S]_{n \geq 0} - \text{IB}$. The factor $\frac{i\kappa^2 H^2}{2^6 \pi^4} \frac{mHaa'}{2} \left(\ln \frac{H^2 \Delta x^2}{4} \right) \sum_{n=0}^{\infty} Y^n$ multiplies all contributions. Here $Y = \frac{y}{4}$; $\ln Y$ and 1 are the multiplicative factors for the each individual row.

I-J _{sub}		$\frac{Ha'}{\Delta x^2}$	$\frac{H^2 a^2 \Delta \eta}{\Delta x^2}$	$\frac{H^2 a^2 \gamma^0 \gamma^k \Delta x_k}{\Delta x^2}$	$\frac{H^2 a a' \gamma^0 \gamma^k \Delta x_k}{\Delta x^2}$	$H^3 a^2 a'$
(1-1)	1	0	$-6(3n^2+6n+2)$	$6(3n^2+6n+2)$	0	$-3(3n^2+8n+5)$
(1-1)	$\ln Y$	0	$-6(n^3+3n^2+2n)$	$6(n^3+3n^2+2n)$	0	$-3(n^3+4n^2+5n+2)$
(1-2)	1	0	0	$-2(3n^2+6n+2)$	0	0
(1-2)	$\ln Y$	0	0	$-2(n^3+3n^2+2n)$	0	0
(2-1)	1	$-2(6n^2+6n+1)$	0	0	$-2(3n^2+4n+1)$	0
(2-1)	$\ln Y$	$-2(2n^3+3n^2+n)$	0	0	$-2(n^3+2n^2+n)$	0
(2-2) _a	1	0	0	0	0	0
(2-2) _a	$\ln Y$	0	0	0	0	0
(2-2) _b	1	0	0	0	0	0
(2-2) _b	$\ln Y$	0	0	0	0	0
(2-2) _c	1	0	0	0	0	0
(2-2) _c	$\ln Y$	0	0	0	0	0
total	1	$-2(6n^2+6n+1)$	$-6(3n^2+6n+2)$	$4(3n^2+6n+2)$	$-2(3n^2+4n+1)$	$-3(3n^2+8n+5)$
total	$\ln Y$	$-2(2n^3+3n^2+n)$	$-6(n^3+3n^2+2n)$	$4(n^3+3n^2+2n)$	$-2(n^3+2n^2+n)$	$-3(n^3+4n^2+5n+2)$

Table 28: $i\delta\Delta_A \times i[S]_{n \geq 0} - \text{IC}$. The factor $\frac{i\kappa^2 H^2}{2^6 \pi^4} \frac{mHaa'}{2} \left(\ln \frac{H^2 \Delta x^2}{4} \right) \sum_{n=0}^{\infty} Y^n$ multiplies all contributions. Here $Y = \frac{y}{4}$; $\ln Y$ and 1 are the multiplicative factors for the each individual row.

I-J		$\frac{\Delta\eta^3}{\Delta x^6}$	$\frac{\gamma^0 \Delta\eta^2 \gamma^k \Delta x_k}{\Delta x^6}$	$\frac{\Delta\eta}{\Delta x^4}$	$\frac{\gamma^0 \gamma^k \Delta x_k}{\Delta x^4}$
(1-1)	1	0	0	$12(3n^2+8n+3)$	$-12(3n^2+8n+3)$
(1-1)	$\ln Y$	0	0	$12(n^3+4n^2+3n)$	$-12(n^3+4n^2+3n)$
(1-2)	1	0	0	0	$4(3n^2+4n+1)$
(1-2)	$\ln Y$	0	0	0	$4(n^3+2n^2+n)$
(1-3)	1	0	0	$-6(2n+1)$	$8(2n+1)$
(1-3)	$\ln Y$	0	0	$-6(n^2+n)$	$8(n^2+n)$
(2-1)	1	$-8(3n^2-2n)$	$8(3n^2-2n)$	$-4(6n^2+2n)$	$4(3n^2+2n+1)$
(2-1)	$\ln Y$	$-8(n^3-n^2)$	$8(n^3-n^2)$	$-4(2n^3+n^2)$	$4(n^3+n^2+n)$
(2-2)	1	$-2(3n^2-2n)$	$-2(3n^2-2n)$	$-(6n^2+2n-3)$	$-(6n^2+6n-1)$
(2-2)	$\ln Y$	$-2(n^3-n^2)$	$-2(n^3-n^2)$	$-(2n^3+n^2-3n)$	$-(2n^3+3n^2-n)$
(2-3)	1	$10(2n-1)$	$-6(2n-1)$	$2(10n-2)$	$-2(2n+1)$
(2-3)	$\ln Y$	$10(n^2-n)$	$-6(n^2-n)$	$2(5n^2-2n)$	$-2(n^2+n)$
(3-1)	1	$-8(2n-2)$	$8(2n-2)$	$2(4n-1)$	$-2(8n-3)$
(3-1)	$\ln Y$	$-8(n^2-2n)$	$8(n^2-2n)$	$2(2n^2-n)$	$-2(4n^2-3n)$
(3-2)	1	$-2(2n-2)$	$-2(2n-2)$	$-(4n-1)$	$(4n-3)$
(3-2)	$\ln Y$	$-2(n^2-2n)$	$-2(n^2-2n)$	$-(2n^2-n)$	$(2n^2-3n)$
(3-3)	1	-5	3	$-\frac{7}{2}$	$-\frac{1}{2}$
(3-3)	$\ln Y$	$-5n$	$3n$	$-\frac{7}{2}n$	$-\frac{1}{2}n$
total	1	$-5(6n^2-4n-1)$	$3(6n^2-4n-1)$	$(6n^2+98n+\frac{49}{2})$	$-(18n^2+78n+\frac{37}{2})$
total	$\ln Y$	$-5(2n^3-2n^2-n)$	$3(2n^3-2n^2-n)$	$(2n^3+49n^2+\frac{49}{2}n)$	$-(6n^3+39n^2+\frac{37}{2}n)$

Table 29: $i\delta\Delta_A \times i[S]_{n \geq 0} - \text{IIA}$. The factor $\frac{i\kappa^2 H^2}{2^6 \pi^4} \frac{mHaa'}{2} \sum_{n=0}^{\infty} Y^n$ multiplies all contributions. Here $Y = \frac{y}{4}$; $\ln Y$ and 1 are the multiplicative factors for the each individual row.

I-J		$\frac{Ha\Delta\eta^2}{\Delta x^4}$	$\frac{Ha'\Delta\eta^2}{\Delta x^4}$	$\frac{Ha\gamma^0\Delta\eta\gamma^k\Delta x_k}{\Delta x^4}$	$\frac{Ha'\gamma^0\Delta\eta\gamma^k\Delta x_k}{\Delta x^4}$	$\frac{Ha}{\Delta x^2}$
(1-1)	1	$12(3n^2+8n+3)$	0	$-12(3n^2+8n+3)$	0	$6(3n^2+6n+2)$
(1-1)	$\ln Y$	$12(n^3+4n^2+3n)$	0	$-12(n^3+4n^2+3n)$	0	$6(n^3+3n^2+2n)$
(1-2)	1	0	0	$4(3n^2+4n+1)$	0	0
(1-2)	$\ln Y$	0	0	$4(n^3+2n^2+n)$	0	0
(1-3)	1	$-6(2n+1)$	0	$8(2n+1)$	0	0
(1-3)	$\ln Y$	$-6(n^2+n)$	0	$8(n^2+n)$	0	0
(2-1)	1	$2(3n^2+4n+1)$	$-2(3n^2-1)$	$-2(3n^2+4n+1)$	$2(3n^2-1)$	$(6n^2+14n+5)$
(2-1)	$\ln Y$	$2(n^3+2n^2+n)$	$-2(n^3-n)$	$-2(n^3+2n^2+n)$	$2(n^3-n)$	$(2n^3+7n^2+5n)$
(2-2)	1	$(3n^2+4n+1)$	0	0	0	$\frac{1}{2}(6n^2+14n+5)$
(2-2)	$\ln Y$	(n^3+2n^2+n)	0	0	0	$\frac{1}{2}(2n^3+7n^2+5n)$
(2-3)	1	$-3(2n+1)$	0	$2(2n+1)$	0	$-3(2n+1)$
(2-3)	$\ln Y$	$-3(n^2+n)$	0	$2(n^2+n)$	0	$-3(n^2+n)$
(3-1)	1	$4n$	$-2(2n+1)$	$-2(2n)$	$2(2n+1)$	$-(2n+3)$
(3-1)	$\ln Y$	$(2n^2-2)$	$-2(n^2+n)$	$-2(n^2-1)$	$2(n^2+n)$	$-(n^2+3n+2)$
(3-2)	1	$6n$	0	$2(2n)$	0	$6n+\frac{9}{2}$
(3-2)	$\ln Y$	$3(n^2-1)$	0	$2(n^2-1)$	0	$3n^2+\frac{9}{2}n+\frac{3}{2}$
(3-3)	1	$\frac{5}{2}$	0	0	0	$\frac{7}{4}$
(3-3)	$\ln Y$	$\frac{5}{2}(n+1)$	0	0	0	$\frac{7}{4}(n+1)$
total	1	$5(9n^2+20n+\frac{13}{2})$	$-2(3n^2+2n)$	$-2(15n^2+34n+12)$	$2(3n^2+2n)$	$27n^2+55n+\frac{79}{4}$
total	$\ln Y$	$5(3n^3+10n^2+\frac{13}{2}n-\frac{1}{2})$	$-2(n^3+n^2)$	$-2(5n^3+17n^2+12n)$	$2(n^3+n^2)$	$9n^3+\frac{55}{2}n^2+\frac{77}{4}n+\frac{5}{4}$

Table 30: $i\delta\Delta_A \times i[S]_{n\geq 0} - \text{IIB}$. The factor $\frac{i\kappa^2 H^2}{2^6 \pi^4} \frac{mHaa'}{2} \sum_{n=0}^{\infty} Y^n$ multiplies all contributions. Here $Y = \frac{y}{4}$; $\ln Y$ and 1 are the multiplicative factors for the each individual row.

I-J		$\frac{Ha'}{\Delta x^2}$	$\frac{H^2 a^2 \Delta \eta}{\Delta x^2}$	$\frac{H^2 a^2 \gamma^0 \gamma^k \Delta x_k}{\Delta x^2}$	$\frac{H^2 a a' \gamma^0 \gamma^k \Delta x_k}{\Delta x^2}$	$H^3 a^2 a'$
(1-1)	1	0	$-3(3n^2+10n+8)$	$3(3n^2+10n+8)$	0	$-\frac{3}{2}(3n^2+8n+5)$
(1-1)	$\ln Y$	0	$-3(n^3+5n^2+8n+4)$	$3(n^3+5n^2+8n+4)$	0	$-\frac{3}{2}(n^3+4n^2+5n+2)$
(1-2)	1	0	0	$-(3n^2+10n+8)$	0	0
(1-2)	$\ln Y$	0	0	$-(n^3+5n^2+8n+4)$	0	0
(1-3)	1	0	$\frac{3}{2}(2n+3)$	$-2(2n+3)$	0	0
(1-3)	$\ln Y$	0	$\frac{3}{2}(n^2+3n+2)$	$-2(n^2+3n+2)$	0	0
(2-1)	1	$-(6n^2+6n+1)$	0	0	$-(3n^2+4n+1)$	0
(2-1)	$\ln Y$	$-(2n^3+3n^2+n)$	0	0	$-(n^3+2n^2+n)$	0
(2-2)	1	0	0	0	0	0
(2-2)	$\ln Y$	0	0	0	0	0
(2-3)	1	0	0	0	0	0
(2-3)	$\ln Y$	0	0	0	0	0
(3-1)	1	$(2n-2)$	0	0	$2(2n+2)$	0
(3-1)	$\ln Y$	(n^2-2n-3)	0	0	$2(n^2+2n+1)$	0
(3-2)	1	0	0	0	0	0
(3-2)	$\ln Y$	0	0	0	0	0
(3-3)	1	0	0	0	0	0
(3-3)	$\ln Y$	0	0	0	0	0
total	1	$-(6n^2+4n+3)$	$-3(3n^2+9n+\frac{13}{2})$	$2(3n^2+8n+5)$	$-(3n^2-3)$	$-\frac{3}{2}(3n^2+8n+5)$
total	$\ln Y$	$-(2n^3+2n^2+3n+3)$	$-3(n^3+\frac{9}{2}n^2+\frac{13}{2}n+3)$	$2(n^3+4n^2+5n+2)$	$-(n^3-3n-2)$	$-\frac{3}{2}(n^3+4n^2+5n+2)$

Table 31: $i\delta\Delta_A \times i[S]_{n \geq 0} - \text{IIC}$. The factor $\frac{i\kappa^2 H^2}{2^6 \pi^4} \frac{mHaa'}{2} \sum_{n=0}^{\infty} Y^n$ multiplies all contributions. Here $Y = \frac{y}{4}$; $\ln Y$ and 1 are the multiplicative factors for the each individual row.

and its various derivatives in four dimensions occurred very frequently,

$$i\delta\Delta_A(x; x') = \frac{-H^2}{8\pi^2} \left[\ln\left(\frac{H^2\Delta x^2}{4}\right) + \frac{1}{2} \right], \quad (163)$$

$$\partial_i i\delta\Delta_A(x; x') = -\partial_i i\delta\Delta_A(x; x') = \frac{-H^2}{4\pi^2} \frac{\Delta x_i}{\Delta x^2}, \quad (164)$$

$$\partial_0 i\delta\Delta_A(x; x') = -\partial'_0 i\delta\Delta_A(x; x') = \frac{H^2}{4\pi^2} \frac{\Delta\eta}{\Delta x^2} \quad (165)$$

$$\partial_0 \partial'_0 i\delta\Delta_A(x; x') = \frac{-H^2}{4\pi^2} \left[\frac{1}{\Delta x^2} + \frac{2\Delta\eta^2}{\Delta x^4} \right], \quad (166)$$

$$\partial_0 \partial_i i\delta\Delta_A(x; x') = -\partial'_0 \partial_i i\delta\Delta_A(x; x') = \frac{H^2}{4\pi^2} \frac{-2\Delta\eta\Delta x_i}{\Delta x^4}, \quad (167)$$

$$\partial_i \partial_l i\delta\Delta_A(x; x') = \frac{H^2}{4\pi^2} \left[\frac{-\delta_{il}}{\Delta x^2} + \frac{2\Delta x_i \Delta x_l}{\Delta x^4} \right]. \quad (168)$$

We also make use of the following identities to facilitate our computation more effectively,

$$\gamma^l \gamma^k \gamma_l = \gamma^k, \quad \gamma^\mu \gamma^0 \gamma_\mu = 2\gamma^0, \quad (169)$$

$$\gamma^l \gamma^k \gamma^0 \gamma^l \Delta x_k = \gamma^0 \gamma^k \Delta x_k, \quad \gamma^k \gamma^l \gamma^0 \gamma^l \Delta x_k = -3\gamma^0 \gamma^k \Delta x_k, \quad (170)$$

$$\gamma^j \gamma^\nu \gamma^0 \gamma^j \Delta x_\nu = 3\Delta\eta + \gamma^0 \gamma^k \Delta x_k, \quad (171)$$

$$\gamma^0 \gamma^\mu \gamma^0 \gamma^\nu \Delta x_\mu \Delta x_\nu = \Delta x^2 - 2\gamma^0 \Delta\eta \gamma^\mu \Delta x_\mu, \quad (172)$$

$$\gamma^k \gamma^\mu \gamma^0 \gamma^\nu \Delta x_k \Delta x_\mu \Delta x_\nu = -\gamma^0 \gamma^k \Delta x_k \Delta x^2 - 2\Delta\eta \gamma^k \Delta x_k \gamma^\mu \Delta x_\mu, \quad (173)$$

$$\gamma^0 \gamma^\nu \gamma^0 \gamma^k \Delta\eta \Delta x_\nu \Delta x_k = \Delta\eta \overline{\Delta x^2} - \gamma^0 \Delta\eta^2 \gamma^k \Delta x_k, \quad (174)$$

$$\gamma^k \gamma^\nu \gamma^0 \gamma^l \Delta x_k \Delta x_l \Delta x_\nu = \Delta\eta \overline{\Delta x^2} - \gamma^0 \gamma^k \Delta x_k \overline{\Delta x^2}. \quad (175)$$

Note that any derivatives acting on $i\delta\Delta_A$ would eliminate $\ln(\frac{H^2\Delta x^2}{4})$ and that the exceptions are the generic contractions (1-1), (1-2), (2-1) and (2-2). We list these results in Table 26, 27 and 28. The rest of the contributions without $\ln \frac{H^2\Delta x^2}{4}$ are summarized in the Table 29, 30 and 31. From these tables one can see that the derivative of the coefficient with $\ln \frac{y}{4}$ is the coefficient without $\ln \frac{y}{4}$. Based on the characteristic of (137) we should not be too surprised at this peculiar pattern occurring here again as in the previous subsection. The final result for $-i[\Sigma^{\text{dAn}0}](x; x')$ could be computed using (142) and (143) and then adding the lowest order constant to each distinct contribution because the summation here starts from $n = 0$ rather than $n = 1$. Finally we tabulate the lengthy results in Table 32 and Table 33.

	$\ln \frac{H^2 \Delta x^2}{4}$	1
$\frac{\Delta \eta^3}{\Delta x^6}$	$-20f_1(Y) - 20f_2(Y) \ln Y$	$-5g_1(Y) - 5g_2(Y) \ln Y$
$\frac{\gamma^0 \Delta \eta^2 \gamma^k \Delta x_k}{\Delta x^6}$	$12f_1(Y) + 12f_2(Y) \ln Y$	$3g_1(Y) + 3g_2(Y) \ln Y$
$\frac{\Delta \eta}{\Delta x^4}$	$2f_3(Y) + 2f_4(Y) \ln Y$	$g_3(Y) + g_4(Y) \ln Y$
$\frac{\gamma^0 \gamma^k \Delta x_k}{\Delta x^4}$	$-6f_5(Y) - 6f_6(Y) \ln Y$	$-g_5(Y) - g_6(Y) \ln Y$
$\frac{Ha \Delta \eta^2}{\Delta x^4}$	$6f_7(Y) + 6f_8(Y) \ln Y$	$5g_7(Y) + 5g_8(Y) \ln Y$
$\frac{Ha' \Delta \eta^2}{\Delta x^4}$	$-4f_9(Y) - 4f_{10}(Y) \ln Y$	$-2g_9(Y) - 2g_{10}(Y) \ln Y$
$\frac{Ha \gamma^0 \Delta \eta \gamma^k \Delta x_k}{\Delta x^4}$	$-4f_{11}(Y) - 4f_{12}(Y) \ln Y$	$-2g_{11}(Y) - 2g_{12}(Y) \ln Y$
$\frac{Ha' \gamma^0 \Delta \eta \gamma^k \Delta x_k}{\Delta x^4}$	$4f_9(Y) + 4f_{10}(Y) \ln Y$	$2g_9(Y) + 2g_{10}(Y) \ln Y$
$\frac{Ha}{\Delta x^2}$	$9f_{13}(Y) + 9f_{14}(Y) \ln Y$	$g_{13}(Y) + g_{14}(Y) \ln Y$
$\frac{Ha'}{\Delta x^2}$	$-2f_{15}(Y) - 2f_{16}(Y) \ln Y$	$-g_{15}(Y) - g_{16}(Y) \ln Y$
$\frac{H^2 a^2 \Delta \eta}{\Delta x^2}$	$-6f_{17}(Y) - 6f_{18}(Y) \ln Y$	$-3g_{17}(Y) - 3g_{18}(Y) \ln Y$
$\frac{H^2 a^2 \gamma^0 \gamma^k \Delta x_k}{\Delta x^2}$	$4f_{17}(Y) + 4f_{18}(Y) \ln Y$	$2g_{19}(Y) + 2g_{20}(Y) \ln Y$
$\frac{H^2 a a' \gamma^0 \gamma^k \Delta x_k}{\Delta x^2}$	$-2f_{19}(Y) - 2f_{20}(Y) \ln Y$	$-g_{21}(Y) - g_{22}(Y) \ln Y$
$H^3 a^2 a'$	$-3f_{21}(Y) - 3f_{22}(Y) \ln Y$	$-\frac{3}{2}g_{19}(Y) - \frac{3}{2}g_{20}(Y) \ln Y$

Table 32: The total result for $i\delta\Delta_A \times i[S]_{n \geq 0}$. The factor $\frac{i\kappa^2 H^2}{2^6 \pi^4} \frac{m H a a'}{2}$ multiplies all contributions. Here $Y = \frac{y}{4}$; $\ln \frac{H^2 \Delta x^2}{4}$ and 1 are the multiplicative factors for the each individual column. The various functions $f_i(Y)$ and $g_i(Y)$ are presented in Table 33

$f_i(Y)$		$g_i(Y)$	
$f_1(Y)$	$\frac{Y(2Y^2+5Y-1)}{(1-Y)^3} + 2$	$g_1(Y)$	$\frac{-Y(Y^2-12Y-1)}{(1-Y)^3} - 1$
$f_2(Y)$	$\frac{6Y^3}{(1-Y)^4}$	$g_2(Y)$	$\frac{Y(3Y^2+10Y-1)}{(1-Y)^4}$
$f_3(Y)$	$\frac{Y(7Y^2-86Y+91)}{(1-Y)^3} + 7$	$g_3(Y)$	$\frac{Y(49Y^2-282Y+257)}{2(1-Y)^3} + \frac{49}{2}$
$f_4(Y)$	$\frac{-6Y(5Y^2+Y-8)}{(1-Y)^4}$	$g_4(Y)$	$\frac{Y(-45Y^2-82Y+151)}{2(1-Y)^4}$
$f_5(Y)$	$\frac{Y(5Y^2-22Y+29)}{(1-Y)^3} + 5$	$g_5(Y)$	$\frac{Y(37Y^2-194Y+229)}{2(1-Y)^3} + \frac{37}{2}$
$f_6(Y)$	$\frac{2Y(-Y^2-Y+8)}{(1-Y)^4}$	$g_6(Y)$	$\frac{Y(-29Y^2-26Y+127)}{2(1-Y)^4}$
$f_7(Y)$	$\frac{Y(3Y^2-7Y+34)}{(1-Y)^3} + 3$	$g_7(Y)$	$\frac{Y(13Y^2-48Y+71)}{2(1-Y)^3} + \frac{13}{2}$
$f_8(Y)$	$\frac{2Y(7Y+8)}{(1-Y)^4}$	$g_8(Y)$	$\frac{Y(Y^3-4Y^2+Y+38)}{2(1-Y)^4} - \frac{1}{2}$
$f_9(Y)$	$\frac{Y(-Y^2+5Y+2)}{(1-Y)^3} - 1$	$g_9(Y)$	$\frac{Y(Y+5)}{(1-Y)^3}$
$f_{10}(Y)$	$\frac{6Y^2}{(1-Y)^4}$	$g_{10}(Y)$	$\frac{2Y(2Y+1)}{(1-Y)^4}$
$f_{11}(Y)$	$\frac{Y(7Y^2-23Y+46)}{(1-Y)^3} + 7$	$g_{11}(Y)$	$\frac{Y(12Y^2-43Y+61)}{(1-Y)^3} + 12$
$f_{12}(Y)$	$\frac{6Y(Y+4)}{(1-Y)^4}$	$g_{12}(Y)$	$\frac{-2Y(2Y-17)}{(1-Y)^4}$
$f_{13}(Y)$	$\frac{Y(3Y^2-10Y+19)}{(1-Y)^3} + 3$	$g_{13}(Y)$	$\frac{Y(79Y^2-270Y+407)}{4(1-Y)^3} + \frac{79}{4}$
$f_{14}(Y)$	$\frac{2Y(Y+5)}{(1-Y)^4}$	$g_{14}(Y)$	$\frac{Y(-5Y^3+20Y^2-29Y+230)}{4(1-Y)^4} + \frac{5}{4}$
$f_{15}(Y)$	$\frac{Y(Y^2-2Y+13)}{(1-Y)^3} + 1$	$g_{15}(Y)$	$\frac{Y(3Y^2-4Y+13)}{(1-Y)^3} + 3$
$f_{16}(Y)$	$\frac{6Y(Y+1)}{(1-Y)^4}$	$g_{16}(Y)$	$\frac{Y(-3Y^3+12Y^2-7Y+10)}{(1-Y)^4} + 3$
$f_{17}(Y)$	$\frac{Y(2Y^2-7Y+11)}{(1-Y)^3} + 2$	$g_{17}(Y)$	$\frac{Y(13Y^2-38Y+37)}{2(1-Y)^3} + \frac{13}{2}$
$f_{18}(Y)$	$\frac{6Y}{(1-Y)^4}$	$g_{18}(Y)$	$\frac{3Y(-Y^3+4Y^2-6Y+5)}{(1-Y)^4} + 3$
$f_{19}(Y)$	$\frac{Y(Y^2-3Y+8)}{(1-Y)^3} + 1$	$g_{19}(Y)$	$\frac{Y(5Y^2-15Y+16)}{(1-Y)^3} + 5$
$f_{20}(Y)$	$\frac{2Y(Y+2)}{(1-Y)^4}$	$g_{20}(Y)$	$\frac{-2Y(Y^3-4Y^2+6Y-6)}{(1-Y)^4} + 2$
$f_{21}(Y)$	$\frac{Y(5Y^2-15Y+16)}{(1-Y)^3} + 5$	$g_{21}(Y)$	$\frac{-3Y^2(Y-3)}{(1-Y)^3} - 3$
$f_{22}(Y)$	$\frac{-2Y(Y^3-4Y^2+6Y-6)}{(1-Y)^4} + 2$	$g_{22}(Y)$	$\frac{2Y(Y^3-4Y^2+8Y-2)}{(1-Y)^4} - 2$

Table 33: The coefficient functions for the table 32

5.3 Sub-Leading Contributions from $i\delta\Delta_B$

In this subsection we work out the contribution from substituting the residual B -type part of the graviton propagator in Table 5,

$$i[\alpha\beta\Delta_{\rho\sigma}] \longrightarrow -[\delta_\alpha^0\delta_\sigma^0\bar{\eta}_{\beta\rho} + \delta_\alpha^0\delta_\rho^0\bar{\eta}_{\beta\sigma} + \delta_\beta^0\delta_\sigma^0\bar{\eta}_{\alpha\rho} + \delta_\beta^0\delta_\rho^0\bar{\eta}_{\alpha\sigma}]i\delta\Delta_B. \quad (176)$$

As in the two previous sub-sections we first make the requisite contractions and then act the derivatives. The result of this first step is summarized in Table 34. We have sometimes broken the result for a single vertex pair into parts because the four different tensors in (176) can make distinct contributions, and because distinct contributions also come from breaking up factors of $\gamma^\alpha J^{\beta\mu}$. These distinct contributions are labeled by subscripts a , b , c , etc.

$i\delta\Delta_B(x; x')$ is the residual of the B -type propagator (67) after the conformal contribution has been subtracted,

$$i\delta\Delta_B(x; x') = \frac{H^2\Gamma(\frac{D}{2})}{16\pi^{\frac{D}{2}}} \frac{(aa')^{2-\frac{D}{2}}}{\Delta x^{D-4}} - \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-2)}{\Gamma(\frac{D}{2})} + \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \sum_{n=1}^{\infty} \left\{ \frac{\Gamma(n+\frac{D}{2})}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} - \frac{\Gamma(n+D-2)}{\Gamma(n+\frac{D}{2})} \left(\frac{y}{4}\right)^n \right\} \quad (177)$$

As was the case for the $i\delta\Delta_A(x; x')$ contributions considered in the previous sub-section, this diagram is not sufficiently singular for the infinite series terms from $i\delta\Delta_B(x; x')$ to make a nonzero contribution in the $D=4$ limit. Unlike $i\delta\Delta_A(x; x')$, even the $n=0$ terms of $i\delta\Delta_B(x; x')$ vanish for $D=4$. Most of the contractions involve at least one derivative of $i\delta\Delta_B$,

$$\partial_i i\delta\Delta_B(x; x') = -\frac{H^2\Gamma(\frac{D}{2})}{16\pi^{\frac{D}{2}}} (D-4)(aa')^{2-\frac{D}{2}} \frac{\Delta x^i}{\Delta x^{D-2}} = -\partial'_i \delta\Delta_B(x; x'), \quad (178)$$

$$\partial_0 i\delta\Delta_B(x; x') = \frac{H^2\Gamma(\frac{D}{2})}{16\pi^{\frac{D}{2}}} (D-4)(aa')^{2-\frac{D}{2}} \left\{ \frac{\Delta\eta}{\Delta x^{D-2}} - \frac{aH}{2\Delta x^{D-4}} \right\}, \quad (179)$$

$$\partial'_0 i\delta\Delta_B(x; x') = \frac{H^2\Gamma(\frac{D}{2})}{16\pi^{\frac{D}{2}}} (D-4)(aa')^{2-\frac{D}{2}} \left\{ -\frac{\Delta\eta}{\Delta x^{D-2}} - \frac{a'H}{2\Delta x^{D-4}} \right\}, \quad (180)$$

$$\partial_0 \partial'_0 i\delta\Delta_B(x; x') = \frac{H^2\Gamma(\frac{D}{2})}{16\pi^{\frac{D}{2}}} \frac{(D-4)}{(aa')^{\frac{D}{2}-2}} \left\{ \frac{-(D-2)\Delta\eta^2}{\Delta x^D} - \frac{1}{\Delta x^{D-2}} \right\} + \mathcal{O}[(D-4)^2], \quad (181)$$

$$\partial_0 \partial_k i\delta\Delta_B(x; x') = -\partial'_0 \partial_k \delta\Delta_B(x; x')$$

I	J	sub	$iV_I^{\alpha\beta}(x) i[S](x; x') iV_J^{\rho\sigma}(x') [\alpha\beta T_{\rho\sigma}^B] i\delta\Delta_B(x; x')$
2	1		0
2	2	a	$-\frac{1}{2}\kappa^2\partial'_0\{\gamma^{(0)}\partial^k i[S](x; x')\gamma_k i\delta\Delta_B(x; x')\}$
2	2	b	$-\frac{1}{2}\kappa^2\partial_k\{\gamma^{(0)}\partial^k i[S](x; x')\gamma^0 i\delta\Delta_B(x; x')\}$
2	3	a	$-\frac{1}{8}\kappa^2\gamma_k\partial_0 i[S](x; x')\gamma^k\partial'_0 i\delta\Delta_B(x; x')$
2	3	b	$\frac{1}{8}\kappa^2\gamma^0\partial'_0 i\delta\Delta_B(x; x')\partial_k i[S](x; x')\gamma^k$
2	3	c	$-\frac{1}{8}\kappa^2\gamma^k\partial_k i\delta\Delta_B(x; x')\partial_0 i[S](x; x')\gamma^0$
2	3	d	$\frac{1}{8}\kappa^2\gamma^0\partial^k i[S](x; x')\gamma^0\partial_k i\delta\Delta_B(x; x')$
3	1		0
3	2	a	$\frac{1}{8}\kappa^2\partial'_0\{\gamma^k i[S](x; x')\gamma_k\partial_0 i\delta\Delta_B(x; x')\}$
3	2	b	$\frac{1}{8}\kappa^2\gamma^k\partial_k\{i[S](x; x')\gamma^0\partial_0 i\delta\Delta_B(x; x')\}$
3	2	c	$-\frac{1}{8}\kappa^2\gamma^0\partial'_0\{i[S](x; x')\gamma^k\partial_k i\delta\Delta_B(x; x')\}$
3	2	d	$-\frac{1}{8}\kappa^2\partial_k\{\gamma^0 i[S](x; x')\gamma^0\partial^k i\delta\Delta_B(x; x')\}$
3	3	a	$-\frac{1}{16}\kappa^2\gamma_k i[S](x; x')\gamma^k\partial_0\partial'_0 i\delta\Delta_B(x; x')$
3	3	b	$\frac{1}{16}\kappa^2\gamma^0 i[S](x; x')\gamma^k\partial_k\partial'_0 i\delta\Delta_B(x; x')$
3	3	c	$-\frac{1}{16}\kappa^2\gamma^k i[S](x; x')\gamma^0\partial_0\partial_k i\delta\Delta_B(x; x')$
3	3	d	$\frac{1}{16}\kappa^2\gamma^0 i[S](x; x')\gamma^0\nabla^2 i\delta\Delta_B(x; x')$

Table 34: Contractions from the $i\delta\Delta_B$ part of the graviton propagator. The contributions from (1-1)–(1-4), (4-1)–(4-4), (2-4) and (3-4) are zero.

pre-factor	$\frac{\kappa^2 H^2}{2^8 \pi^D} \Gamma(\frac{D}{2}) \Gamma(\frac{D}{2} - 1) \frac{ma}{(aa')^{\frac{D}{2}-2}}$	$\frac{\kappa^2 H^{D-2}}{2^{D+4} \pi^D} \frac{\Gamma(D-2) \Gamma(\frac{D-1}{2})}{\Gamma(\frac{D}{2})} ma$
$(2-2)_{a_1}$	$(D-1) [\frac{(D-2)}{2(D-3)} \partial_0^2 + Ha \partial_0] \frac{1}{\Delta x^{2D-6}}$	$-(D-1) [\partial_0^2 + Ha \partial_0] \frac{1}{\Delta x^{D-2}}$
$(2-2)_{a_2}$	$[\frac{(D-2)}{2(D-3)} \gamma^0 \partial_0 \bar{\partial} - Ha \gamma^0 \bar{\partial}] \frac{1}{\Delta x^{2D-6}}$	$[\gamma^0 \partial_0 \bar{\partial} + Ha \gamma^0 \bar{\partial}] \frac{1}{\Delta x^{D-2}}$
$(2-2)_{b_1}$	$\frac{(D-2)}{2(D-3)} \gamma^0 \partial_0 \bar{\partial} \frac{1}{\Delta x^{2D-6}}$	$-\gamma^0 \partial_0 \bar{\partial} \frac{1}{\Delta x^{D-2}}$
$(2-2)_{b_2}$	$\frac{-(D-2)}{2(D-3)} \nabla^2 \frac{1}{\Delta x^{2D-6}}$	$\nabla^2 \frac{1}{\Delta x^{D-2}}$

Table 35: The contributions for the contraction (2-2) from $i\delta\Delta_B \times i[S]_{\text{fm}}$.

$$= \frac{H^2 \Gamma(\frac{D}{2})}{16\pi^{\frac{D}{2}}} \frac{(D-4)(D-2)}{(aa')^{\frac{D}{2}-2}} \left\{ \frac{-\Delta\eta\Delta x_k}{\Delta x^D} \right\} + \mathcal{O}[(D-4)^2], \quad (182)$$

$$\nabla^2 i\delta\Delta_B(x; x') = \frac{H^2 \Gamma(D) 2}{16\pi^{\frac{D}{2}}} \frac{(D-4)}{(aa')^{\frac{D}{2}-2}} \left\{ \frac{(D-2)\overline{\Delta x^2}}{\Delta x^D} - \frac{(D-1)}{\Delta x^{D-2}} \right\}. \quad (183)$$

The fact that the first line of $i\delta\Delta_B(x; x')$ and its various derivatives are all of the order $(D-4)$ means that they would only contribute when they are multiplied by a divergence. Note that the contractions consisting of mass interaction vertices all vanish through (176), so no order m contributions come from $i\delta\Delta_B \times i[S]_{\text{cf}}$. The potential non-zero, order m contributions could either come from the flat spacetime mass term or from the infinite series expansion of the fermion propagator. Remember that the only term in $i[S]_{\text{fm}}$ behaves like $\frac{1}{\Delta x^{D-2}}$ and that the most singular one in $i[S]_{n \geq 0}$ goes like $\frac{1}{\Delta x^{D-3}}$. In addition, the generic contractions in Table 34 are comprised of two derivatives. Therefore one can count that the dimensionality of most singular contribution from $i[S]_{\text{fm}}$ is $\frac{1}{\Delta x^{2D-4}}$ whereas the one from $i[S]_{n \geq 0}$ is $\frac{1}{\Delta x^{2D-5}}$. The former is logarithmically divergent in $D = 4$ before performing the partial integration, so one still needs to keep arbitrary D for the computation; the latter is entirely integrable in $D = 4$ so that one could compute this part in four dimensions and the result turns out to be zero owing to the cancelation of the first two series of $i\delta\Delta_B(x; x')$ and owing to $(D-4)$ factor from its various derivatives. Therefore the only class we need to work out in this sub-section is $i\delta\Delta_B(x; x') \times i[S]_{\text{fm}}(x; x')$.

We take special care of the contraction (2-2) because it is the only contraction in Table 34 which derivatives might have a chance not to act upon $i\delta\Delta_B$. We also break up $\gamma^{(0)}\partial^k$ into $\frac{1}{2}\gamma^0\partial^k$ and $\frac{1}{2}\gamma^k\partial^0$ for each sub-contraction and

$I - J_{\text{sub}}$	contributions
$(2-3)_a$	$\frac{(D-1)}{2(D-3)} [\partial^2 - (D-4)\partial_0^2] \frac{1}{\Delta x^{2D-6}}$
$(2-3)_b$	$-\frac{(D-4)}{2(D-3)} \gamma^0 \partial_0 \bar{\partial} \frac{1}{\Delta x^{2D-6}}$
$(2-3)_c$	$\frac{(D-4)}{2(D-3)} \gamma^0 \partial_0 \bar{\partial} \frac{1}{\Delta x^{2D-6}}$
$(2-3)_d$	$\frac{(D-1)}{2(D-3)} [\partial^2 + (D-4)(D-1)\nabla^2] \frac{1}{\Delta x^{2D-6}}$
$(3-2)_a$	$\frac{(D-4)(D-1)}{(D-3)} \partial_0^2 \frac{1}{\Delta x^{2D-6}}$
$(3-2)_b$	$-\frac{(D-4)}{(D-3)} \gamma^0 \partial_0 \bar{\partial} \frac{1}{\Delta x^{2D-6}}$
$(3-2)_c$	$\frac{(D-4)}{(D-3)} \gamma^0 \partial_0 \bar{\partial} \frac{1}{\Delta x^{2D-6}}$
$(3-2)_d$	$-\frac{(D-4)}{(D-3)} \nabla^2 \frac{1}{\Delta x^{2D-6}}$
$(3-3)_a$	$\frac{-(D-1)}{4(D-3)} \partial^2 \frac{1}{\Delta x^{2D-6}}$
$(3-3)_b$	$\frac{-(D-4)}{4(D-3)} \gamma^0 \partial_0 \bar{\partial} \frac{1}{\Delta x^{2D-6}}$
$(3-3)_c$	$\frac{(D-4)}{4(D-3)} \gamma^0 \partial_0 \bar{\partial} \frac{1}{\Delta x^{2D-6}}$
$(3-3)_d$	$[\frac{-(D-1)}{4(D-3)} \partial^2 + \frac{(D-4)}{4(D-3)} \nabla^2] \frac{1}{\Delta x^{2D-6}}$
total	$\left\{ \frac{(D-1)}{2(D-3)} \partial^2 + (D-4) \left[\frac{(D-1)}{(D-3)} \partial_0^2 \right] - \frac{3}{4(D-3)} \nabla^2 \right\} \frac{1}{\Delta x^{2D-6}}$

Table 36: $i\delta\Delta_B \times i[S]_{\text{fm}}$ terms. All contributions are multiplied by $\frac{\kappa^2 H^2}{2^{10}\pi^D} ma\Gamma(\frac{D}{2})\Gamma(\frac{D}{2} - 1)(aa')^{2-\frac{D}{2}}$.

tabulate the results in Table 35. These expressions are integrable in four dimensions and the contributions from the left column cancel out exactly with that from the right column in $D = 4$. Table 36 gives the rest of our results for the most singular contributions, those in which all derivatives act upon the conformal coordinate separation. There is no net contribution when one or more of the derivatives acts upon a scale factor. Those expressions are also integrable in $D = 4$ dimensions, at which point we can take $D = 4$ and the result vanishes on account of the overall factor of $(D - 4)$ or $(D - 4)^2$. Finally we read off the net contribution from Table 36 and take $D = 4$,

$$-i[\Sigma^{\text{idBfm}}](x; x') = \frac{\kappa^2 H^2}{2^{10} \pi^4} ma \left\{ \frac{3}{2} \partial^2 \frac{1}{\Delta x^2} \right\} = \frac{\kappa^2 H^2}{2^8 \pi^4} ma \left\{ \frac{3}{8} \partial^2 \frac{1}{\Delta x^2} \right\}. \quad (184)$$

5.4 Sub-Leading Contributions from $i\delta\Delta_C$

The point of this subsection is to compute the contribution from replacing the graviton propagator in Table 5 by its residual C -type part,

$$i[\alpha\beta\Delta_{\rho\sigma}] \rightarrow 2 \left[\frac{\bar{\eta}_{\alpha\beta}\bar{\eta}_{\rho\sigma}}{(D-2)(D-3)} + \frac{\delta_\alpha^0\delta_\beta^0\bar{\eta}_{\rho\sigma} + \bar{\eta}_{\alpha\beta}\delta_\rho^0\delta_\sigma^0}{D-2} + \left(\frac{D-3}{D-2}\right)\delta_\alpha^0\delta_\beta^0\delta_\rho^0\delta_\sigma^0 \right] i\delta\Delta_C. \quad (185)$$

As in the previous sub-sections we first make the requisite contractions and then act the derivatives. The result of this first step is summarized in Table 37 and Table 38. We have sometimes broken the result for a single vertex pair into parts because the four different tensors in (185) can make distinct contributions, and because distinct contributions also come from breaking up factors of $\gamma^\alpha J^{\beta\mu}$. These distinct contributions are labeled by subscripts a , b , c , etc.

Here $i\delta\Delta_C(x; x')$ is the residual of the C -type propagator (68) after the conformal contribution has been subtracted,

$$i\delta\Delta_C(x; x') = \frac{H^2}{16\pi^{\frac{D}{2}}} \left(\frac{D}{2} - 3 \right) \Gamma\left(\frac{D}{2} - 1\right) \frac{(aa')^{2-\frac{D}{2}}}{\Delta x^{D-4}} + \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-3)}{\Gamma(\frac{D}{2})} \\ - \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \sum_{n=1}^{\infty} \left\{ \left(n - \frac{D}{2} + 3 \right) \frac{\Gamma(n + \frac{D}{2} - 1)}{\Gamma(n+2)} \left(\frac{y}{4} \right)^{n - \frac{D}{2} + 2} (n+1) \frac{\Gamma(n+D-3)}{\Gamma(n + \frac{D}{2})} \left(\frac{y}{4} \right)^n \right\}. \quad (186)$$

As with the contributions from $i\delta\Delta_B(x; x')$ considered in the previous sub-section, the only way $i\delta\Delta_C(x; x')$ can give a nonzero contribution in $D = 4$

I	J	sub	$iV_I^{\alpha\beta}(x) i[S](x; x') iV_J^{\rho\sigma}(x') [\alpha_\beta T_{\rho\sigma}^C] i\delta\Delta_C(x; x')$
1	4		$\frac{2}{(D-3)(D-2)} \kappa^2 iam \bar{\partial} i[S](x; x') i\delta\Delta_C(x; x')$
2	4	a	$\frac{1}{(D-2)} \kappa^2 iam \gamma^0 \partial_0 i[S](x; x') i\delta\Delta_C(x; x')$
2	4	b	$-\frac{1}{(D-3)(D-2)} \kappa^2 iam \bar{\partial} i[S](x; x') i\delta\Delta_C(x; x')$
3	4	a	$\frac{(D-1)}{2(D-3)(D-2)} \kappa^2 iam \gamma^0 \partial_0 i\delta\Delta_C(x; x') i[S](x; x')$
3	4	b	$\frac{1}{2(D-3)(D-2)} \kappa^2 iam \bar{\partial} i\delta\Delta_C(x; x') i[S](x; x')$
4	1		$-\frac{2}{(D-3)(D-2)} \kappa^2 iam \partial'_\mu \{i[S](x; x') \gamma^\mu i\delta\Delta_C(x; x')\}$
4	2	a	$-\frac{1}{(D-2)} \kappa^2 iam \partial'_0 \{i[S](x; x') \gamma^0 i\delta\Delta_C(x; x')\}$
4	2	b	$-\frac{1}{(D-3)(D-2)} \kappa^2 iam \partial_k \{i[S](x; x') \gamma^k i\delta\Delta_C(x; x')\}$
4	3	a	$\frac{(D-1)}{2(D-3)(D-2)} \kappa^2 iam i[S](x; x') \gamma^0 \partial_0 i\delta\Delta_C(x; x')$
4	3	b	$\frac{1}{2(D-3)(D-2)} \kappa^2 iam i[S](x; x') \bar{\partial} i\delta\Delta_C(x; x')$

Table 37: Contractions from the $i\delta\Delta_C$ part of the graviton propagator-I.

dimensions is for it to multiply a singular term. That means only the $n=0$ term can possibly contribute. Even for the $n=0$ term, both derivatives must act upon the coordinate separation to make a nonzero contribution in $D=4$ dimensions.

All the vertex pairs involve one or more derivatives of $i\delta\Delta_C$,

$$\partial_i i\delta\Delta_C = \frac{H^2 \Gamma(\frac{D}{2}-1)}{16\pi^{\frac{D}{2}}} (\frac{D}{2}-3)(D-4)(aa')^{2-\frac{D}{2}} \frac{-\Delta x^i}{\Delta x^{D-2}} = -\partial'_i i\delta\Delta_C, \quad (187)$$

$$\partial_0 i\delta\Delta_C = \frac{H^2 \Gamma(\frac{D}{2}-1)}{16\pi^{\frac{D}{2}}} (\frac{D}{2}-3)(D-4)(aa')^{2-\frac{D}{2}} \left\{ \frac{\Delta\eta}{\Delta x^{D-2}} - \frac{aH}{2\Delta x^{D-4}} \right\}, \quad (188)$$

$$\partial'_0 i\delta\Delta_C = \frac{H^2 \Gamma(\frac{D}{2}-1)}{16\pi^{\frac{D}{2}}} (\frac{D}{2}-3)(D-4)(aa')^{2-\frac{D}{2}} \left\{ -\frac{\Delta\eta}{\Delta x^{D-2}} - \frac{a'H}{2\Delta x^{D-4}} \right\}, \quad (189)$$

$$\begin{aligned} \partial_0 \partial'_0 i\delta\Delta_C = \frac{H^2 \Gamma(\frac{D}{2}-1)}{16\pi^{\frac{D}{2}}} (\frac{D}{2}-3) \frac{(D-4)}{(aa')^{\frac{D}{2}-2}} \left\{ \frac{-(D-2)\Delta\eta^2}{\Delta x^D} - \frac{1}{\Delta x^{D-2}} \right\} \\ + \mathcal{O}[(D-4)^2], \end{aligned} \quad (190)$$

$$\partial_k \partial_0 i\delta\Delta_C = -\partial_k \partial'_0 i\delta\Delta_C$$

I	J	sub	$iV_I^{\alpha\beta}(x) i[S](x; x') iV_J^{\rho\sigma}(x') [\alpha_\beta T_{\rho\sigma}^C] i\delta\Delta_C(x; x')$
1	1		$-\frac{2}{(D-3)(D-2)}\kappa^2\partial'_\mu\{\partial i[S](x; x')\gamma^\mu i\delta\Delta_C(x; x')\}$
1	2	a	$-\frac{1}{(D-2)}\kappa^2\partial'_0\{\partial i[S](x; x')\gamma^0 i\delta\Delta_C(x; x')\}$
1	2	b	$-\frac{1}{(D-3)(D-2)}\kappa^2\partial_k\{\partial i[S](x; x')\gamma^k i\delta\Delta_C(x; x')\}$
1	3	a	$\frac{(D-1)}{2(D-3)(D-2)}\kappa^2\partial i[S](x; x')\gamma^0\partial'_0 i\delta\Delta_C(x; x')$
1	3	b	$-\frac{1}{2(D-3)(D-2)}\kappa^2\partial i[S](x; x')\bar{\partial} i\delta\Delta_C(x; x')$
2	1	a	$-\frac{1}{(D-2)}\kappa^2\partial'_\mu\{\gamma^0\partial_0 i[S](x; x')\gamma^\mu i\delta\Delta_C(x; x')\}$
2	1	b	$\frac{1}{(D-3)(D-2)}\kappa^2\partial'_\mu\{\bar{\partial} i[S](x; x')\gamma^\mu i\delta\Delta_C(x; x')\}$
2	2	a	$-\frac{(D-3)}{2(D-2)}\kappa^2\partial'_0\{\gamma^0\partial_0 i[S](x; x')\gamma^0 i\delta\Delta_C(x; x')\}$
2	2	b	$-\frac{1}{2(D-2)}\kappa^2\partial_k\{\gamma^0\partial_0 i[S](x; x')\gamma^k i\delta\Delta_C(x; x')\}$
2	2	c	$\frac{1}{2(D-2)}\kappa^2\partial'_0\{\bar{\partial} i[S](x; x')\gamma^0 i\delta\Delta_C(x; x')\}$
2	2	d	$\frac{1}{2(D-3)(D-2)}\kappa^2\partial_k\{\bar{\partial} i[S](x; x')\gamma^k i\delta\Delta_C(x; x')\}$
2	3	a	$\frac{1}{4}\left(\frac{D-1}{D-2}\right)\kappa^2\gamma^0\partial_0 i[S](x; x')\gamma^0\partial'_0 i\delta\Delta_C(x; x')$
2	3	b	$-\frac{1}{4(D-2)}\kappa^2\gamma^0\partial_0 i[S](x; x')\bar{\partial} i\delta\Delta_C(x; x')$
2	3	c	$-\frac{(D-1)}{4(D-3)(D-2)}\kappa^2\bar{\partial} i[S](x; x')\gamma^0\partial'_0 i\delta\Delta_C(x; x')$
2	3	d	$\frac{1}{4(D-3)(D-2)}\kappa^2\bar{\partial} i[S](x; x')\bar{\partial} i\delta\Delta_C(x; x')$
3	1	a	$-\frac{(D-1)}{2(D-3)(D-2)}\kappa^2\partial'_\mu\{\gamma^0\partial_0 i\delta\Delta_C(x; x')i[S](x; x')\gamma^\mu\}$
3	1	b	$-\frac{1}{2(D-3)(D-2)}\kappa^2\partial'_\mu\{\bar{\partial} i\delta\Delta_C(x; x')i[S](x; x')\gamma^\mu\}$
3	2	a	$-\frac{1}{4}\left(\frac{D-1}{D-2}\right)\kappa^2\partial'_0\{\gamma^0\partial_0 i\delta\Delta_C(x; x')i[S](x; x')\gamma^0\}$
3	2	b	$-\frac{1}{4(D-2)}\kappa^2\partial'_0\{\bar{\partial} i\delta\Delta_C(x; x')i[S](x; x')\gamma^0\}$
3	2	c	$-\frac{(D-1)}{4(D-3)(D-2)}\kappa^2\partial_k\{\gamma^0\partial_0 i\delta\Delta_C(x; x')i[S](x; x')\gamma^k\}$
3	2	d	$-\frac{1}{4(D-3)(D-2)}\kappa^2\partial_k\{\bar{\partial} i\delta\Delta_C(x; x')i[S](x; x')\gamma^k\}$
3	3	a	$\frac{(D-1)^2}{8(D-3)(D-2)}\kappa^2\gamma^0 i[S](x; x')\gamma^0\partial_0\partial'_0 i\delta\Delta_C(x; x')$
3	3	b	$-\frac{(D-1)}{8(D-3)(D-2)}\kappa^2\gamma^0 i[S](x; x')\gamma^k\partial_0\partial_k i\delta\Delta_C(x; x')$
3	3	c	$\frac{(D-1)}{8(D-3)(D-2)}\kappa^2\gamma^k i[S](x; x')\gamma^0\partial_k\partial'_0 i\delta\Delta_C(x; x')$
3	3	d	$-\frac{1}{8(D-3)(D-2)}\kappa^2\gamma^k i[S](x; x')\gamma^l\partial_k\partial_l i\delta\Delta_C(x; x')$

Table 38: Contractions from the $i\delta\Delta_C$ part of the graviton propagator-II.

$$= \frac{H^2 \Gamma(\frac{D}{2}-1)}{16\pi^{\frac{D}{2}}} \left(\frac{D}{2}-3\right) \frac{(D-2)(D-4)}{(aa')^{\frac{D}{2}-2}} \frac{-\Delta\eta\Delta x_k}{\Delta x^D} + \mathcal{O}[(D-4)^2], \quad (191)$$

$$\partial_k \partial_l i\delta\Delta_C = \frac{H^2 \Gamma(\frac{D}{2}-1)}{16\pi^{\frac{D}{2}}} \left(\frac{D}{2}-3\right) \frac{(D-4)}{(aa')^{\frac{D}{2}-2}} \left\{ \frac{(D-2)\Delta x_k \Delta x_l}{\Delta x^D} - \frac{\delta_{kl}}{\Delta x^{D-2}} \right\}. \quad (192)$$

Note that $i\delta\Delta_C$ and its various derivatives have the same behaviors as $i\delta\Delta_B$. The propagator itself tends to cancel in $D = 4$ dimensions and its various derivatives all carry $(D-4)$ factor. This means that they could give the non-zero contributions only when they are multiplied by the singular terms. For the generic contraction I in Table 37, the only order m contribution must come from the conformal part of the fermion propagator whereas the generic contraction II in Table 38 the order m contribution could be either from the flat spacetime mass term or from the infinite series expansion of the fermion propagator. The fortunate thing is that the contribution from the infinite series expansion of the fermion propagator vanishes. The most singular terms from this particular contribution have dimensionality $\frac{1}{\Delta x^{2D-5}}$, which are integrable in $D = 4$. Therefore they completely vanish in $D = 4$ dimensions owing to the behaviors of the residual part of the C-type graviton propagator we mentioned above.

We first work out the contributions from Table 37, which contain a logarithmic divergence. The contraction $(1-4)$ and $(2-4)_a$ are simple owing to the special property of the conformal part of the fermion propagator (103) and hence they only pick up the constant part of $i\delta\Delta_C$,

$$[1-4] = \frac{\kappa^2 H^{D-2}}{2^D \pi^{\frac{D}{2}}} \frac{\Gamma(D-3)}{\Gamma(\frac{D}{2})} \frac{2}{(D-2)(D-3)} i m a \delta^D(x-x'), \quad (193)$$

$$[2-4]_a = \frac{\kappa^2 H^{D-2}}{2^D \pi^{\frac{D}{2}}} \frac{\Gamma(D-3)}{\Gamma(\frac{D}{2})} \frac{1}{(D-2)} i m a \delta^D(x-x'). \quad (194)$$

The same procedure is employed to make the expressions integrable in $D = 4$. We summarized the contractions $(4-1)$, $(4-2)_a$ and $(4-2)_b$ in Table 39 and listed the rest of this category in Table 40. The two series in Table 39 from each contraction cancel out with each other precisely in four dimensions. Finally the total summation from (193), (194), Table 39 and Table 40 is quite simple in $D = 4$,

$$-i[\Sigma^{\text{idCcf}}](x; x') = \frac{i\kappa^2 H^2}{16\pi^2} \frac{3}{2} m a \delta^4(x-x') + \frac{\kappa^2 H^2}{32\pi^4} m a \left\{ \frac{-9}{16} \partial^2 \frac{1}{\Delta x^2} \right\}. \quad (195)$$

I-J _{sub}	$\frac{\kappa^2 H^2}{2^5 \pi^D} \Gamma(\frac{D}{2}) \Gamma(\frac{D}{2} - 1) \frac{(3-\frac{D}{2})}{(D-3)} \frac{ma}{(aa')^{\frac{D}{2}-2}}$	$\frac{\kappa^2 H^{D-2}}{2^{D+1} \pi^D} \Gamma(D-3) ma$
(4-1)	$\frac{-1}{(D-2)(D-3)} \partial^2 \frac{1}{\Delta x^{2D-6}}$	$\frac{2}{(D-2)^2(D-3)} \partial^2 \frac{1}{\Delta x^{D-2}}$
(4-2) _a	$\frac{1}{2(D-2)} \not{\partial} \gamma^0 \partial_0 \frac{1}{\Delta x^{2D-6}}$	$\frac{-1}{(D-2)^2} \not{\partial} \gamma^0 \partial_0 \frac{1}{\Delta x^{D-2}}$
(4-2) _b	$\frac{-1}{2(D-2)(D-3)} \not{\partial} \bar{\not{\partial}} \frac{1}{\Delta x^{2D-6}}$	$\frac{1}{(D-2)^2(D-3)} \not{\partial} \bar{\not{\partial}} \frac{1}{\Delta x^{D-2}}$

Table 39: Contributions from Table 37 for $i\delta\Delta_C \times i[S]_{\text{cf}}(x; x')$ -I.

I-J _{sub}	$\partial^2 \frac{1}{\Delta x^{2D-6}}$
(2-4) _b	$\frac{-1}{4}$
(3-4) _a	$\frac{-1}{8}$
(3-4) _b	$\frac{-1}{8}$
(4-3) _a	$\frac{-1}{8}$
(4-3) _b	$\frac{-1}{8}$

Table 40: Contributions from Table 37 for $i\delta\Delta_C \times i[S]_{\text{cf}}(x; x')$ -II. All the terms are multiplied by $\frac{\kappa^2 H^2}{2^5 \pi^D} \Gamma(\frac{D}{2}) \Gamma(\frac{D}{2} - 1) \frac{(3-\frac{D}{2})(D-1)}{(D-2)^2(D-3)^2} \frac{ma}{(aa')^{\frac{D}{2}-2}}$.

I-J _{sub}	$\frac{\kappa^2 H^2}{2^6 \pi^D} \Gamma^2(\frac{D}{2}-1)(3-\frac{D}{2}) \frac{ma}{(aa')^{\frac{D}{2}-2}}$	$\frac{\kappa^2 H^{D-2}}{2^{D+2} \pi^D} \Gamma(D-3) ma \frac{2}{(D-2)}$
(1-1)	$[\frac{1}{(D-3)^2} \partial^2 - \frac{2Ha\gamma^0}{(D-2)(D-3)} \not{\partial}] \frac{1}{\Delta x^{2D-6}}$	$\frac{2[-\partial^2 + Ha\gamma^0 \not{\partial}]}{(D-2)(D-3)} \frac{1}{\Delta x^{D-2}}$
(1-2) _a	$[\frac{-1}{2(D-3)} \not{\partial} \gamma^0 \partial_0 - \frac{Ha}{(D-2)} \partial_0] \frac{1}{\Delta x^{2D-6}}$	$\frac{[\not{\partial} \gamma^0 \partial_0 + Ha \partial_0]}{(D-2)} \frac{1}{\Delta x^{D-2}}$
(1-2) _b	$[\frac{1}{2(D-3)^2} \not{\partial} + \frac{Ha\gamma^0}{(D-2)(D-3)}] \bar{\not{\partial}} \frac{1}{\Delta x^{2D-6}}$	$\frac{-[\not{\partial} + Ha\gamma^0]}{(D-2)(D-3)} \bar{\not{\partial}} \frac{1}{\Delta x^{D-2}}$
(2-1) _a	$[\frac{-1}{2(D-3)} \gamma^0 \partial_0 - \frac{Ha\gamma^0}{(D-2)}] \not{\partial} \frac{1}{\Delta x^{2D-6}}$	$\frac{[\gamma^0 \partial_0 + Ha\gamma^0]}{(D-2)} \not{\partial} \frac{1}{\Delta x^{D-2}}$
(2-1) _b	$\frac{1}{2(D-3)^2} \bar{\not{\partial}} \not{\partial} \frac{1}{\Delta x^{2D-6}}$	$\frac{-1}{(D-2)(D-3)} \bar{\not{\partial}} \not{\partial} \frac{1}{\Delta x^{D-2}}$
(2-2) _a	$[-\frac{1}{4} \partial_0^2 - \frac{(D-3)}{2(D-2)} Ha \partial_0] \frac{1}{\Delta x^{2D-6}}$	$\frac{(D-3)}{2(D-2)} [\partial_0^2 + Ha \partial_0] \frac{1}{\Delta x^{D-2}}$
(2-2) _b	$[\frac{1}{4(D-3)} \gamma^0 \partial_0 + \frac{Ha\gamma^0}{2(D-2)}] \bar{\not{\partial}} \frac{1}{\Delta x^{2D-6}}$	$\frac{-[\gamma^0 \partial_0 + Ha\gamma^0]}{2(D-2)} \bar{\not{\partial}} \frac{1}{\Delta x^{D-2}}$
(2-2) _c	$\frac{-1}{4} \gamma^0 \partial_0 \bar{\not{\partial}} \frac{1}{\Delta x^{2D-6}}$	$\frac{1}{2(D-2)} \gamma^0 \partial_0 \bar{\not{\partial}} \frac{1}{\Delta x^{D-2}}$
(2-2) _d	$\frac{1}{4(D-3)^2} \nabla^2 \frac{1}{\Delta x^{2D-6}}$	$\frac{-1}{2(D-2)(D-3)} \nabla^2 \frac{1}{\Delta x^{D-2}}$

Table 41: Contributions from Table 38 for $i\delta\Delta_C \times i[S]_{\text{fm}}(x; x')$ -I.

I-J _{sub}	$\partial^2 \frac{1}{\Delta x^{2D-6}}$
(1-3) _a	$\frac{-1}{2(D-3)}$
(1-3) _b	$\frac{-1}{2(D-3)}$
(2-3) _a	$\frac{-1}{4}$
(2-3) _d	$\frac{1}{4(D-3)}$
(3-3) _a	$\frac{1}{8}(\frac{D-1}{D-3})$
(3-3) _d	$\frac{1}{8(D-3)}$
total	$\frac{(D-8)}{8(D-3)} - \frac{(D-4)}{4(D-3)}$

Table 42: Contribution from Table 38 for $i\delta\Delta_C \times i[S]_{\text{fm}}(x; x')$ -II. All the terms are multiplied by $\frac{\kappa^2 H^2}{2^8 \pi^D} \Gamma^2(\frac{D}{2}-1) \frac{(3-\frac{D}{2})(D-1)}{(D-2)(D-3)} \frac{ma}{(aa')^{\frac{D}{2}-2}}$.

The final class we need to complete is the contributions from Table 38. Very similarly to what happened with $i\delta\Delta_B$, the contractions from (1-1), (1-2), (2-1), (2-2) tend to cancel and we summarized them in Table 41. We also tabulate the rest of the contributions which do not vanish in $D = 4$ in Table 42. As already explained, terms for which one or more derivative acts upon a scale factor make no contribution in $D = 4$ dimensions, so the final nonzero contribution come from the derivatives only acting upon the coordinate separation, Δx^2 . The net contributions from Table 41 do vanish completely in $D = 4$ dimensions and the only non-zero contributions of this class come from Table 42,

$$-i[\Sigma^{\text{idCfm}}](x; x') = \frac{\kappa^2 H^2}{2^8 \pi^4} m a \left\{ \frac{-3}{4} \partial^2 \frac{1}{\Delta x^2} \right\}. \quad (196)$$

6 Renormalization

Except for the finite results from (144), and the results from Table 33-32, each of which possesses distinctive expressions, the rest of regulated result we have worked so hard to compute derives from summing expressions (98), (114), (117), (124), (126), (136), (154), (162), (184), (195) and (196)

$$\begin{aligned} & i\kappa^2 \left\{ \beta_1 \frac{m}{a} \partial^2 + \beta_2 m H \partial_0 + \beta_3 m H \gamma^0 \bar{\partial} + \beta_4 m H^2 a \right\} \delta^D(x-x') + \frac{i\kappa^2}{16\pi^2} \left\{ \left(3 \ln a - \frac{3}{8} \right) \frac{m}{a} \partial^2 \right. \\ & + \left(\frac{97}{16} \ln a - \frac{63}{16} \right) m H \partial_0 + \left(\frac{9}{16} \ln a + \frac{1}{8} \right) m H \gamma^0 \bar{\partial} + \left(\frac{95}{8} \ln a + \frac{195}{32} \right) H^2 m a \left. \right\} \delta^4(x-x') \\ & + \frac{\kappa^2}{64\pi^4} \left\{ \left[\frac{3}{2} \frac{m}{a'} \partial^2 + \left(\frac{7}{8} \frac{a}{a'} - \frac{27}{32} \right) m H \partial_0 + \left(\frac{9}{16} \frac{a}{a'} - \frac{9}{32} \right) m H \gamma^0 \bar{\partial} + H^2 m \left(\frac{215}{32} a + \right. \right. \right. \\ & \left. \left. \left. \frac{9}{32} a' \right) \right] \partial^2 + H^2 m a \left[\nabla^2 + 6 H a \partial_0 + 4 H a \gamma^0 \bar{\partial} \right] \right\} \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] + \frac{\kappa^2 H^2}{64\pi^4} \left\{ \frac{9}{16} m a' \partial_0^2 \right. \\ & + \left(\frac{-53}{16} m a + \frac{3}{16} m a' \right) \gamma^0 \partial_0 \bar{\partial} + \left(\frac{-49}{16} + \ln \frac{H^2}{4\mu^2} \right) m a \nabla^2 + \left(\frac{9}{2} + 6 \ln \frac{H^2}{4\mu^2} \right) H m a^2 \partial_0 \\ & + \left(\frac{35}{8} - \frac{11}{8} \ln \frac{y}{4} + 4 \ln \frac{H^2}{4\mu^2} \right) H m a^2 \gamma^0 \bar{\partial} + \left(\frac{-5}{8} - \frac{3}{8} \ln \frac{y}{4} \right) H m a a' \gamma^0 \bar{\partial} \left. \right\} \frac{1}{\Delta x^2}. \quad (197) \end{aligned}$$

The various D -dependent constants in (197) are,

$$\beta_1 = \frac{\mu^{D-4}}{16\pi^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2}-1)}{(D-3)(D-4)} \left\{ \frac{-2(D-1)}{(D-2)} \right\}, \quad (198)$$

$$\beta_2 = \frac{\mu^{D-4}}{16\pi^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2}-1)}{(D-3)(D-4)} \left\{ \frac{-4(D-1)}{(D-2)} + (D-2)(b_2+b_3) \right\}, \quad (199)$$

$$\beta_3 = \frac{\mu^{D-4}}{16\pi^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2}-1)}{(D-3)(D-4)} \left\{ \frac{-(D-1)}{2} + (D-2)(b_{2a}+b_{3a}) + \frac{(D-2)}{2}d_1 \right\}, \quad (200)$$

$$\begin{aligned} \beta_4 = & \frac{\mu^{D-4}}{16\pi^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2}-1)}{(D-3)(D-4)} \left\{ \frac{(D-1)}{2} + (D-2)(b_4-b_2) + \frac{(D-2)(d_2+d_3+d_4)}{2} \right. \\ & + \frac{D(D-1)}{8(D-3)} \left[-D(D-2) - \frac{1}{4} - \frac{1}{4}(D-4) \right] \Big\} + \frac{H^{D-4}}{2^D \pi^{\frac{D}{2}} (D-4)} \left\{ \frac{-\Gamma(D+1)}{2\Gamma(\frac{D}{2})} \right. \\ & \left. + \frac{(2D-3)}{4} \Gamma(\frac{D}{2}) \Gamma(\frac{D}{2}-1) \Gamma(3-\frac{D}{2}) + \frac{\Gamma(D)}{\Gamma(\frac{D}{2})} \frac{2}{(D-3)} \right\}. \end{aligned} \quad (201)$$

Here $b_2, b_{2a}, b_3, b_{3a}, b_4, d_1, d_2, d_3$ and d_4 are defined in (118) and (125). In obtaining these expressions we have always chosen to convert finite, $D=4$ terms with ∂^2 acting on $1/\Delta x^2$, into delta functions,

$$\partial^2 \left[\frac{1}{\Delta x^2} \right] = i4\pi^2 \delta^4(x-x'). \quad (202)$$

All such terms have then been included in those which are proportional to $\delta^4(x-x')$.

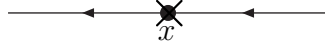


Fig. 3: Contribution from counterterms.

The local divergences in this expression are canceled by the BPHZ counterterms enumerated at the end of section 3. The generic diagram topology is depicted in Fig. 3, and the analytic form is,

$$-i \left[\Sigma^{\text{ctm}} \right] (x; x') = \sum_{I=1}^4 i C_{Iij} \delta^D(x-x'), \quad (203)$$

$$= i\kappa^2 \left\{ \alpha_1 \frac{m}{a} \partial^2 + \alpha_2 m H \partial_0 + \alpha_3 m H \gamma^0 \bar{\not{\partial}} + \alpha_4 H^2 m a \right\} \delta^D(x-x'). \quad (204)$$

In comparing (197) and (204) it would seem that the simplest choice for the coefficients α_i is,

$$\alpha_1 = 3F_1, \quad \alpha_2 = \frac{97}{16}F_1, \quad \alpha_3 = \frac{9}{16}F_1, \quad \text{and} \quad \alpha_4 = \frac{85}{8}F_2. \quad (205)$$

$$\text{Here} \quad F_2 = \frac{\mu^{D-4}}{16\pi^{\frac{D}{2}}(D-4)}, \quad F_1 = \frac{\Gamma(\frac{D}{2}-1)}{(D-3)} \times F_2. \quad (206)$$

Except for the distinctive contributions from (144), Table 33 and Table 32, the rest of our final result for the renormalized self-energy is,

$$\begin{aligned}
-i[\Sigma^{\text{ren}}](x; x') = & \frac{i\kappa^2}{16\pi^2} \left\{ \left[3 \ln a + \frac{1}{8} \right] \frac{m}{a} \partial^2 + \left[\frac{97}{16} \ln a - \frac{39}{16} \right] mH \partial_0 + \left[\frac{9}{16} \ln a \right. \right. \\
& + \left. \frac{5}{16} \right] mH \gamma^0 \bar{\not{\partial}} + \left[\frac{95}{8} \ln a - \frac{29}{32} - \frac{85}{16} \psi(1) + \frac{5}{8} \ln \frac{H^2}{4\mu^2} \right] H^2 ma \left. \right\} \delta^4(x-x') + \frac{\kappa^2}{64\pi^4} \left\{ \right. \\
& \left[\frac{3m}{2a'} \partial^2 + \left(\frac{7a}{8a'} - \frac{27}{32} \right) mH \partial_0 + \left(\frac{9a}{16a'} - \frac{9}{32} \right) mH \gamma^0 \bar{\not{\partial}} + H^2 m \left(\frac{215}{32} a + \frac{9}{32} a' \right) \right] \partial^2 \\
& + H^2 ma \left[\nabla^2 + 6Ha \partial_0 + 4Ha \gamma^0 \bar{\not{\partial}} \right] \left. \right\} \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] + \frac{\kappa^2 H^2}{64\pi^4} \left\{ \frac{9}{16} ma' \partial_0^2 \right. \\
& + \left[\frac{-53}{16} ma + \frac{3}{16} ma' \right] \gamma^0 \partial_0 \bar{\not{\partial}} + \left[\frac{-49}{16} + \ln \frac{H^2}{4\mu^2} \right] ma \nabla^2 + \left[\frac{9}{2} + 6 \ln \frac{H^2}{4\mu^2} \right] Hma^2 \partial_0 \\
& + \left[\frac{35}{8} - \frac{11}{8} \ln \frac{y}{4} + 4 \ln \frac{H^2}{4\mu^2} \right] Hma^2 \gamma^0 \bar{\not{\partial}} + \left[\frac{-5}{8} - \frac{3}{8} \ln \frac{y}{4} \right] Hma a' \gamma^0 \bar{\not{\partial}} \left. \right\} \frac{1}{\Delta x^2}. \quad (207)
\end{aligned}$$

7 Discussion

We have used dimensional regularization to compute quantum gravitational corrections to the fermion self-energy at one loop order in a locally de Sitter background. Our regulated result is (197). Although Dirac + Einstein is not perturbatively renormalizable [5] we obtained a finite result (207) by absorbing the divergences with BPHZ counterterms.

For this 1PI function, and at one loop order, only four counterterms are necessary. None of them represents redefinitions of terms in the Lagrangian of Dirac + Einstein. Two of the required counterterm operators (85) come from generally coordinate invariant fermion bilinears of dimension six (73). The other two counterterm operators (86) are from other fermion bilinears of dimension six (84) which respect the symmetries (44-49) of our de Sitter noninvariant gauge (43).

Although our renormalized result could be changed by altering the finite parts of the three BPHZ counterterms, this does not affect its leading behavior in the far infrared. It is simple to be quantitative about this. Were we to make finite shifts $\Delta\alpha_i$ in our counterterms (205) the induced change in the

renormalized self-energy would be,

$$-i[\Delta\Sigma^{\text{ren}}](x; x') = i\kappa^2 \left\{ \Delta\alpha_1 \frac{m}{a} \partial^2 + \Delta\alpha_2 m H \partial_0 + \Delta\alpha_3 m H \gamma^0 \bar{\not{\partial}} + \alpha_4 H^2 m a \right\} \delta^4(x - x'). \quad (208)$$

No physical principle seems to fix the $\Delta\alpha_i$ so any result that derives from their values is arbitrary. This is why BPHZ renormalization does not yield a complete theory. However, at late times (which accesses the far infrared because all momenta are redshifted by $a(t) = e^{Ht}$) the local part of the renormalized self-energy (207) is dominated by the large logarithms,

$$\frac{i\kappa^2}{16\pi^2} \left\{ 3 \ln a \frac{m}{a} \partial^2 + \frac{97 \ln a}{16} m H \partial_0 + \frac{9 \ln a}{16} m H \gamma^0 \bar{\not{\partial}} + \frac{95 \ln a}{8} H^2 m a \right\} \delta^4(x - x'). \quad (209)$$

The coefficients of these logarithms are finite and completely fixed by our calculation. As long as the shifts $\Delta\alpha_i$ are finite, their impact (208) must eventually be dwarfed by the large logarithms (209).

None of this should seem surprising, although it does with disturbing regularity. The comparison we have just made is a standard feature of low energy effective field theory and has a very old and distinguished pedigree [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 51, 52]. Loops of massless particles make finite, nonanalytic contributions which cannot be changed by local counterterms and which dominate the far infrared. Further, these effects must occur as well, *with precisely the same numerical values*, in whatever fundamental theory ultimately resolves the ultraviolet problem of quantum gravity. That is why Weinberg and Sucher got exactly the same long range force from the exchange of massless neutrinos using Fermi theory [12, 13] as one would get from the Standard Model [13].

So we can use (207) reliably in the far infrared. The point of this exercise has been to study the effect of breaking conformal invariance with small fermion mass. Obtaining (207) completes the first part in that program. What remains is to use our result to solve the quantum-corrected Dirac equation (1). We shall undertake that in a subsequent paper. However, it seems clear that the dominant effect must come from the terms which possess large logarithms in local terms and in (144), Table 32 and Table 33¹⁵.

¹⁵Without an explicit calculation we cannot determine whether or not the $\ln(\frac{y}{4})$ in the non-local terms will produce infrared enhancements because the same term occurring in [4] fails to give them.

As adumbrated in the Introduction, these terms are only enhanced by a factor of $\ln(a)$ relative to the classical part of the Dirac equation (1). That seems roughly the same as the $\ln(a)$ enhancement which soft virtual gravitons induce on massless fermions [1, 2]. Note that any such effect will be independent of assumptions about the existence and couplings of light scalars during inflation.

We have already commented on the importance of the logarithm terms. During inflation these infrared logarithms are ubiquitous in quantum corrections from massless, minimally coupled scalars and gravitons. It is not even possible to exclude the possibility that infrared logarithms can contaminate the power spectrum of cosmological density perturbations [86, 87, 88]! The proportional correction they make in that case must be small because the logarithms would only start to grow at horizon crossing, and must cease growing when the mode reenters the horizon after inflation. So the largest enhancement for a currently observable mode would be $\ln(a) \lesssim 100$. This must be set against the tiny loop counting parameter of $GH^2 \lesssim 10^{-10}$.

The more significant corrections would be to modes which are still enormously super-horizon. These are also down by the constant GH^2 , but the time-dependent enhancement factor $\ln(a)$ could be arbitrarily big. That is what we shall study in our follow-up work. Of course loops of such modes can also engender large corrections to effective interactions of low dimension. These corrections can grow so large that perturbation theory eventually breaks down. Starobinskiĭ has advocated gaining quantitative control over this regime by summing the leading infrared logarithms at each order [89]. With Yokoyama he has given a complete solution for the case of a minimally coupled scalar with arbitrary potential which is a spectator to de Sitter inflation [90]. This powerful non-perturbative technique has been successfully generalized to Yukawa theory [33], which showed that the system decays in a Big Rip singularity, and to SQED [82], which confirmed the conjecture by Davis, Dimopoulos, Prokopec and Tornkvist that super-horizon photons acquire mass during inflation.

The asymptotic late time effect is small in the simple scalar models and in SQED for which the leading logarithm expansion have been summed. However, the same kind of effect derived from Yukawa is huge. Therefore it is by no means clear what might be the outcome for more complicated theories¹⁶

¹⁶The theory possesses derivative interaction vertices which cannot be avoided by imposing a special gauge as was done in SQED.

that also show infrared logarithms such as quantum gravity [70, 71, 72]. Another application of our result (207) is to serve as “data” in checking the validity of the new, more general rule [92, 84] for reproducing the leading logarithms of massive Dirac + Einstein. This might serve as an important intermediate point in the difficult task of generalizing Starobinskiĭ’s techniques to full blown quantum gravity.

It is well to close with a comment on accuracy. Although parts of this computation are quite intricate we have good confidence that (207) is correct for two reasons. First, there is the flat space limit of taking H to zero while taking the conformal time to be $\eta = -e^{-Ht}/H$ with t held fixed. This checks the leading conformal contributions. Our second reason for confidence is the fact that all divergences can be absorbed using just the four counterterms we have inferred in section 3 on the basis of symmetry. This was by no means the case for individual terms; many separate pieces must be added to eliminate other divergences.

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A The reduced fermion propagator and its related identities

Here we listed some identities we have used for various gamma functions contracted with the first derivative of the $n = 0$ part of the fermion propagator (122).

$$\begin{aligned} \not{\partial} i[S]_{n=0} &= \Gamma\left(\frac{D}{2}-1\right) \left\{ \frac{mHaa'}{4\pi^{\frac{D}{2}}} \left[\frac{-\gamma^0}{\Delta x^{D-2}} - \frac{Ha\gamma^0\Delta\eta}{\Delta x^{D-2}} - \frac{H^2a^2\gamma^0}{(D-4)\Delta x^{D-4}} \right] \right. \\ &\quad \left. - \frac{mH^{D-3}}{(4\pi)^{\frac{D}{2}}} (aa')^{\frac{D}{2}-1} \Gamma\left(\frac{D}{2}+1\right) \Gamma\left(2-\frac{D}{2}\right) H^2a^2\gamma^0 \right\}, \quad (210) \\ \partial_\mu i[S]\gamma^\mu &= \Gamma\left(\frac{D}{2}-1\right) \left\{ \frac{mHaa'}{4\pi^{\frac{D}{2}}} \left[\frac{(D-2)\Delta\eta\gamma^\mu\Delta x_\mu}{\Delta x^D} + \frac{Ha\gamma^\mu\Delta x_\mu}{\Delta x^{D-2}} \right] \right. \end{aligned}$$

$$-\frac{H^2 a^2 \gamma^0}{(D-4)\Delta x^{D-4}} \Big] - \frac{mH^{D-3}}{(4\pi)^{\frac{D}{2}}} (aa')^{\frac{D}{2}-1} \Gamma\left(\frac{D}{2}+1\right) \Gamma\left(2-\frac{D}{2}\right) H^2 a^2 \gamma^0 \Big\}. \quad (211)$$

To facilitate the calculation from the infinite series expansion of the fermion propagator for $D = 4$, we might employ the following identities,

$$\begin{aligned} \not{\partial} i[S] = & \frac{mHaa'}{16\pi^2} \sum_{n=0}^{\infty} \left(\frac{y}{4}\right)^n \left\{ \left[\frac{-4\gamma^0}{\Delta x^2} - \frac{4Ha\gamma^0\Delta\eta}{\Delta x^2} \right] \left[(2n+1) + (n^2+n) \ln \frac{y}{4} \right] \right. \\ & \left. + H^2 a^2 \delta_{\mu}^0 \left[(2n+3) + (n^2+3n+2) \ln \frac{y}{4} \right] \right\}, \end{aligned} \quad (212)$$

$$\begin{aligned} \partial_{\mu} i[S] \gamma^{\mu} = & \frac{mHaa'}{16\pi^2} \sum_{n=0}^{\infty} \left(\frac{y}{4}\right)^n \left\{ \frac{4\gamma^0}{\Delta x^2} \left[2n + n^2 \ln \frac{y}{4} \right] - \frac{8\Delta\eta\gamma^{\mu}\Delta x_{\mu}}{\Delta x^4} \left[(2n-1) + (n^2-n) \ln \frac{y}{4} \right] \right. \\ & \left. + \frac{4Ha\gamma^{\mu}\Delta x_{\mu}}{\Delta x^2} \left[(2n+1) + (n^2+n) \ln \frac{y}{4} \right] + H^2 a^2 \delta_{\mu}^0 \left[(2n+3) + (n^2+3n+2) \ln \frac{y}{4} \right] \right\}, \end{aligned} \quad (213)$$

$$\begin{aligned} \bar{\partial} i[S] = & \frac{mHaa'}{16\pi^2} \sum_{n=0}^{\infty} \left(\frac{y}{4}\right)^n \left\{ -4 \left[\frac{\Delta\eta\gamma^k\Delta x_k}{\Delta x^4} + \frac{\gamma^0\overline{\Delta x}^2}{\Delta x^4} \right] \left[(2n-1) + (n^2-n) \ln \frac{y}{4} \right] \right. \\ & \left. - \frac{6\gamma^0}{\Delta x^2} \left[1 + n \ln \frac{y}{4} \right] + \frac{2Ha\gamma^k\Delta x_k}{\Delta x^2} \left[(2n+1) + (n^2+n) \ln \frac{y}{4} \right] \right\}, \end{aligned} \quad (214)$$

$$\begin{aligned} \partial_k i[S] \gamma^k = & \frac{mHaa'}{16\pi^2} \sum_{n=0}^{\infty} \left(\frac{y}{4}\right)^n \left\{ 4 \left[\frac{-\Delta\eta\gamma^k\Delta x_k}{\Delta x^4} + \frac{\gamma^0\overline{\Delta x}^2}{\Delta x^4} \right] \left[(2n-1) + (n^2-n) \ln \frac{y}{4} \right] \right. \\ & \left. + \frac{6\gamma^0}{\Delta x^2} \left[1 + n \ln \frac{y}{4} \right] + \frac{2Ha\gamma^k\Delta x_k}{\Delta x^2} \left[(2n+1) + (n^2+n) \ln \frac{y}{4} \right] \right\}. \end{aligned} \quad (215)$$

The formulae keep the same form for the summation starting from $n = 1$ as long as we are working in $D = 4$.

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